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ABSTRACT

Contemporary Mathematics in Context is a four-year, integrated mathematics program developed by the Core-Plus Mathematics Project (CPMP) at the Everyday Learning Corporation. The program features student and teacher materials for a three-year core curriculum for all students as well as a fourth-year course continuing the preparation of students for college mathematics. The materials were designed to implement the vision of high school mathematics as portrayed in the National Council of Teachers of Mathematics (NCTM) Curriculum and Evaluation Standards for School Mathematics and Professional Standards for Teaching Mathematics. This booklet provides an overview of the curriculum, presents information about the implementation the curriculum, and offers suggestions for managing the classroom activities and assessment. (ASK)

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Implementing the Core-Plus Mathematics Curriculum

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Contemporary Mathematics in Context

A Unified Approach

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CORE-PLUS MATHEMATICS PROJECT

Implementing the Core-Plus Mathematics Curriculum

Contemporary Mathematics in Context

A Unified Approach

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Implementing the Core-Plus Mathematics Curriculum

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A Unified Approach

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About the *Core-Plus Mathematics Project*

The **Core-Plus Mathematics Project (CPMP)** was funded by the National Science Foundation to develop student and teacher materials for a complete high school mathematics curriculum. Courses 1–3 comprise a core curriculum appropriate for *all* students. Course 4 continues the preparation of students for college mathematics.

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Core-Plus Mathematics Project
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Curriculum Overview

INTRODUCTION

Contemporary Mathematics in Context is a four-year integrated mathematics program developed by the **Core-Plus Mathematics Project (CPMP).** The program includes student and teacher materials for a three-year core curriculum for all students, and for a fourth-year course continuing the preparation of students for college mathematics. The materials were designed to implement the vision of high school mathematics portrayed in the National Council of Teachers of Mathematics *Curriculum and Evaluation Standards for School Mathematics* (1989) and *Professional Standards for Teaching Mathematics* (1991).

The curriculum builds upon the theme of *mathematics as sense-making.* Through investigations of real-life contexts, students develop a rich understanding of important mathematics that makes sense to them and which, in turn, enables them to make sense out of new situations and problems. The curriculum materials have the following features:

- **Multiple Connected Strands:** Each year the curriculum features four strands of mathematics, unified by fundamental themes, by common topics, and by habits of mind or ways of thinking.
- **Mathematical Modeling:** The curriculum emphasizes mathematical modeling and modeling concepts, including data collection, representation, interpretation, prediction, and simulation.
- **Access:** The curriculum is designed so that core topics are accessible to all students. Differences in student performance and interest can be accommodated by the depth and level of abstraction to which common topics are pursued, by the nature and degree of difficulty of applications, and by opportunities for student choice on homework tasks and projects.
- **Technology:** Numerical, graphics, and programming link capabilities such as those found on many graphics calculators are assumed and capitalized on.
- **Active Learning:** Instruction and assessment practices are designed to promote mathematical thinking through the use of engaging problem situations. Both collaborative group and individual work are used as students explore, conjecture, verify, apply, evaluate, and communicate mathematical ideas.

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A curriculum with the above features promises to make mathematics accessible to a diverse student population. Developing mathematics each year along multiple strands nurtures the differing strengths and talents of students and simultaneously helps them to develop diverse mathematical insights. Developing mathematics from a modeling perspective permits students to experience mathematics as a means of making sense of data and problems that arise in diverse contexts within and across cultures. Engaging students in small groups to work together on tasks develops their ability both to deal with, and find commonality in, diversity of ideas. Using calculators as a means for learning and doing mathematics enables students to develop versatile ways of dealing with realistic situations and reduces the manipulative skill filter that has prevented large numbers of students from continuing their study of significant mathematics. In addition, calculator graphics offer powerful, easily understood new ways of visualizing mathematics across each of the strands.

Each course in the *Contemporary Mathematics in Context* program has gone through at least a three-year research, development, and evaluation cycle. This cycle includes pilot testing of the complete course and then revision followed by national field testing and further refinement. Course 1 was field-tested during the 1994–95 school year with approximately 5,650 students in 36 high schools in Alaska, California, Colorado, Georgia, Idaho, Iowa, Kentucky, Michigan, Ohio, South Carolina, and Texas. Courses 2 and 3 were field tested in the same schools with approximately 3,710 and 2,660 students, respectively. Represented was a broad and diverse cross section of students from urban, suburban, and rural communities. Students successfully using the curriculum have ranged from special education students in mainstreamed classes to highly motivated and talented students in multidistrict mathematics and science centers.

UNIFIED MATHEMATICS

Each course of *Contemporary Mathematics in Context* features "strands" of algebra and functions, geometry and trigonometry, statistics and probability, and discrete mathematics. These four strands are connected within units by fundamental ideas such as symmetry, recursion, function, data analysis, and curve-fitting. The strands also are connected across units by mathematical habits of mind such as visual thinking, recursive thinking, searching for and describing patterns, making and checking conjectures, reasoning with multiple representations, inventing mathematics, and providing convincing arguments. The strands are linked further by the fundamental themes of data, representation, shape, and change. Important mathematical ideas are continually revisited through this attention to connections within and across strands, enabling students to develop a robust understanding of mathematics.

Algebra and Functions

The algebra and functions strand develops students' ability to recognize, represent, and solve problems involving relations among quantitative variables. Central to the development is the use of functions as mathematical models. The key algebraic models in the curriculum are linear, exponential, power, and periodic functions and combinations of those basic types. Each algebraic model is investigated in at least three linked representations—graphic, numeric, and symbolic—with the aid of technology. Attention is also given to modeling with systems, both linear and nonlinear, and to symbolic reasoning.

Geometry and Trigonometry

The primary goal of the geometry and trigonometry strand is to develop visual thinking and students' ability to construct, reason with, interpret, and apply mathematical models of patterns in the visual world. Modeling visual patterns involves describing the patterns with regard to shape, size, and location; representing visual patterns with drawings, coordinates or vectors; and predicting changes, and invariants in visual patterns when some force is applied or action occurs to an object. Drawing, constructing, manipulating, and analyzing models of visual patterns in two and three dimensions are emphasized.

Statistics and Probability

The primary role of the statistics and probability strand is to develop students' ability to analyze data intelligently, to recognize and measure variation, and to understand the patterns that underlie probabilistic situations. Graphical methods of data analysis, simulations, sampling, and experience with the collection and interpretation of real data are featured.

Discrete Mathematics

The discrete mathematics strand develops students' ability to model and solve problems involving sequential change, decision-making in finite settings, and relationships among a finite number of elements. Topics include matrices, vertex-edge graphs, recursion, models of social decision making, and systematic counting methods. Key themes are existence *(Is there a solution?)*, optimization *(What is the best solution?)*, and algorithmic problem solving *(Can you efficiently construct a solution?)*.

ORGANIZATION OF THE CURRICULUM

Each course of the *Contemporary Mathematics in Context* curriculum consists of seven units and a culminating capstone experience. Each of the units is comprised of three to six multiday lessons in which major ideas are developed through investigations of rich applied problems. The time needed to complete a unit varies from approximately three to six weeks. The final element of each course, the capstone, is a thematic, two-week, project-oriented activity that enables students to pull together and apply the important mathematical concepts and methods developed in the entire course. Unit titles for the three-year core curriculum for all students are given in the following table. Goals and mathematical topics for each of these units are on pages 6 through 11.

SCOPE AND SEQUENCE FOR THREE-YEAR CORE PROGRAM

Unit	Course 1	Unit	Course 2	Unit	Course 3
1	Patterns in Data	1	Matrix Models	1	Multiple-Variable Models
2	Patterns of Change	2	Patterns of Location, Shape, and Size	2	Modeling Public Opinion
3	Linear Models	3	Patterns of Association	3	Symbol Sense and Algebraic Reasoning
4	Graph Models	4	Power Models	4	Shapes and Geometric Reasoning
5	Patterns in Space and Visualization	5	Network Optimization	5	Patterns in Variation
6	Exponential Models	6	Geometric Form and Its Function	6	Families of Functions
7	Simulation Models	7	Patterns in Chance	7	Discrete Models of Change
CAPSTONE	Planning a Benefits Carnival	CAPSTONE	Forests, the Environment, and Mathematics	CAPSTONE	Making the Best of It: Optimal Forms and Strategies

In developing *Contemporary Mathematics in Context*, the Core-Plus Mathematics Project chose mathematical content the developers believe is the most important mathematics all high-school students should have the opportunity to learn. In particular, the content of the last units in the texts is not viewed as optional as is often the case with traditional textbooks. The developers believe that this content is so important and broadly useful that it should be completed by students before they embark on the next course of the *Contemporary Mathematics in Context* program. This is the primary reason why each course is available in a two-part edition. A school whose student population does not complete a course in a single year can use Part B of the two-part edition at the start of the next academic year, while new students begin with Part A.

The organization of the student texts differs in several other ways from traditional textbooks. There are no boxed-off definitions, "worked out" examples, or content summaries. Instead, students learn mathematics by doing mathematics. Concept images are developed as students complete investigations, and later concept definitions may appear. Mathematical ideas are developed and then shared by groups of students at strategically placed "Checkpoints" within the lessons. These discussions then lead to a class summary of shared understandings.

It is important that each student construct a Math Toolkit consisting of mathematical concepts and methods as they are developed. Blackline masters are provided for students to organize their toolkits by strand and then by concepts/definitions/algorithms and relationships/properties. By organizing these important class-generated ideas from a unit, from occasional summary "Checkpoint" responses, or from a unit summary in the student's own words, the student will gather a valuable set of tools that can be used throughout the course and in subsequent courses.

Research, development, and classroom evaluation of a flexible fourth-year course for college-bound students is in progress. As indicated in the chart below, Course 4 will consist of a core of four units for all college-bound students, plus sequences of specialized units based on intended undergraduate major.

COURSE 4 SCOPE AND SEQUENCE

Core Units		Additional Units Leading to Mathematical and Physical Sciences		Additional Units Leading to Social, Management, and Health Sciences	
1	Modeling Motion	5A	Polynomial and Rational Functions	5B	Binomial Distributions
2	Rates of Change	6A	Functions and Symbolic Reasoning	6B	Problem Solving, Algorithms, and Spreadsheets
3	Counting Models	7A	Space Geometry	7B	Informatics
4	Logarithmic Functions and Data Models	8A	Mathematical Structures	8B	Designs of Experiments
		CAPSTONE	Building Mathematical Bridges	CAPSTONE	Analyzing Published Reports

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OVERVIEW OF COURSE 1

Unit 1 **Patterns in Data** develops student ability to make sense out of real-world data through use of graphical displays and summary statistics.

Topics include distributions of data and their shapes, as displayed in number line plots, histograms, box plots, and stem-and-leaf plots; scatterplots and association; plots over time and trends; measures of center, including mean, median, and mode and their properties; measures of variation, including percentiles, interquartile range, and mean absolute deviation and their properties; and transformations of data.

Unit 2 **Patterns of Change** develops student ability to recognize important patterns of change among variables and to represent those patterns using tables of numerical data, coordinate graphs, verbal descriptions, and symbolic rules.

Topics include coordinate graphs, tables, algebraic formulas (rules), relationships between variables, linear functions, nonlinear functions, and NOW-NEXT recurrence relations.

Unit 3 **Linear Models** develops student confidence and skill in using linear equations to model and solve problems in situations which exhibit constant (or nearly constant) rate of change or slope.

Topics include linear functions, slope of a line, rate of change, intercepts, distributive property, linear equations (including $y = a + bx$ and NOW-NEXT forms), solving linear equations and inequalities, using linear equations to model given data, and determining best-fit lines for scatterplot data.

Unit 4 **Graph Models** develops student ability to use vertex-edge graphs to represent and analyze real-world situations involving relationships among a finite number of elements, including scheduling, managing conflicts, and finding efficient routes.

Topics include vertex-edge graph models, optimization, algorithmic problem solving, Euler circuits and paths, matrix representation of graphs, graph coloring, chromatic number, digraphs, and critical path analysis.

Unit 5 **Patterns in Space and Visualization** develops student visualization skills and an understanding of properties of space-shapes, including symmetry, area, and volume.

Topics include two- and three-dimensional shapes, spatial visualization, perimeter, area, surface area, volume, the Pythagorean Theorem, polygons and their properties, symmetry, isometric transformations (reflections, rotations, translations, glide reflections), one-dimensional strip patterns, tilings of the plane, and the regular (Platonic) solids.

Unit 6 **Exponential Models** develops student ability to use exponential functions to model and solve problems in situations which exhibit exponential growth or decay.

Topics include exponential growth, exponential functions, fractals, exponential decay, recursion, half-life, compound growth, finding equations to fit exponential patterns in data, and properties of exponents.

CONTINUED

Unit 7 **Simulation Models** develops student confidence and skill in using simulation methods—particularly those involving the use of random numbers—to make sense of real-world situations involving chance.

Topics include simulation, frequency tables and their histograms, random-digit tables and random-number generators, independent events, Law of Large Numbers, and expected number of successes in a series of binomial trials.

CAPSTONE **Looking Back at Course 1: Planning a Benefits Carnival** is a thematic, two-week, project-oriented activity that enables students to pull together and apply the important mathematical concepts and methods developed throughout the course.

The *Contemporary Mathematics in Context* curriculum and the instructional and assessment practices it promotes address the focal points of the National Council of Teachers of Mathematics' *Curriculum Standards* for grades 9–12. By design, the **process standards** for Problem Solving, Reasoning, Communication, and Connections are an integral part of each lesson of every unit in the curriculum.

The chart below correlates Course 1 units with the **content standards** for grades 9–12 in terms of primary emphases (✔) and connections (+).

CORRELATION OF COURSE 1 TO NCTM STANDARDS

Course 1 Units	NCTM Grades 9–12 Content Standards	Algebra	Functions	Geometry from a Synthetic Viewpoint	Geometry from an Algebraic Viewpoint	Trigonometry	Statistics	Probability	Discrete Mathematics	Conceptual Underpinnings of Calculus	Mathematical Structure
Patterns in Data		+	+				✔		+		
Patterns of Change		✔	✔	+			+		✔	+	+
Linear Models		✔	✔	+			✔		+	+	+
Graph Models		+	+	+					✔		+
Patterns in Space and Visualization		✔	+	✔	✔		+		+	+	
Exponential Models		✔	✔	+			✔	+	+	✔	+
Simulation Models		+	+	+			+	✔	+		
Capstone—Planning a Benefits Carnival		✔	✔	✔			✔	✔	✔	✔	

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OVERVIEW OF COURSE 2

Unit 1 **Matrix Models** extends student ability to use matrices and matrix operations to represent and solve problems from a variety of real-world settings while connecting important mathematical ideas from several strands.

Topics include: Matrix models in such areas as inventory control, social relations, archeology, recidivism, ecosystems, sports, tournament rankings, and Markov processes; matrix operations, including row sums, matrix addition, scalar multiplication, matrix multiplication, and matrix powers; properties of matrices; and matrix methods for solving systems of linear equations.

Unit 2 **Patterns of Location, Shape, and Size** develops student understanding of coordinate methods for representing and analyzing relations among geometric shapes and for describing geometric change.

Topics include: Modeling situations with coordinates, including computer-generated graphics, distance, midpoint of a segment, and slope; designing and programming algorithms, matrices, systems of equations, coordinate models of isometric transformations (reflections, rotations, translations, glide reflections) and of size transformations, and similarity.

Unit 3 **Patterns of Association** develops student understanding of the strength of association between two variables, how to measure the degree of the relation, and how to use this measure as a tool to create and interpret prediction lines for paired data.

Topics include: Rank correlation, Pearson's correlation coefficient, cause and effect related to correlation, impact of outliers on correlation, least squares linear models, the relation of correlation to linear models, and variability in prediction.

Unit 4 **Power Models** develops student ability to recognize data patterns that involve both direct and inverse power variation, to construct and analyze those models and combinations such as quadratic functions, and to apply those models to a variety of problems.

Topics include: Basic power models with rules of the form $y = ax^b$ (for positive and negative integers b), combinations of power models with other simple models, analysis of quadratic functions and equations from tabular, graphic, and symbolic viewpoints, square root and cube root relations, and fractional power and radical expressions.

Unit 5 **Network Optimization** extends student ability to use vertex-edge graphs to represent and analyze real-world situations involving network optimization, including optimal spanning networks and shortest routes.

Topics include: Vertex-edge graph models, optimization, algorithmic problem solving, matrices, trees, minimal spanning trees, shortest paths, Hamiltonian circuits and paths, and traveling salesperson problems.

Unit 6 **Geometric Form and Its Function** develops student ability to model and analyze physical phenomena with triangles, quadrilaterals, and circles and to use these shapes to investigate trigonometric functions, angular velocity, and periodic change.

Topics include: Parallelogram linkages, pantographs, similarity, triangular linkages (with one side that can change length); sine, cosine, and tangent ratios, indirect measurement; angular velocity, transmission factor, linear velocity; periodic change; radian measure; period, amplitude, and graphs of functions of the form $y = A\sin(Bx)$, $y = A\cos(Bx)$.

CONTINUED

Unit 7 **Patterns in Chance** develops student ability to understand and visualize situations involving chance by using simulation and mathematical analysis to construct probability distributions.

Topics include: Probability distributions and their graphs, multiplication rule for independent events, geometric distribution, expected value, rare events, summation notation, and an introduction to the binomial distribution.

CAPSTONE **Looking Back at Course 2: Forests, the Environment, and Mathematics** is a thematic, two-week, project-oriented activity that enables students to pull together and apply the important mathematical concepts and methods developed throughout the course.

The *Contemporary Mathematics in Context* curriculum and the instructional and assessment practices it promotes address the focal points of the National Council of Teachers of Mathematics' *Curriculum Standards* for grades 9–12. By design, the **process standards** for Problem Solving, Reasoning, Communication, and Connections are an integral part of each lesson of every unit in the curriculum.

The chart below correlates Course 2 units with the **content standards** for grades 9–12 in terms of primary emphases (✔) and connections (+).

CORRELATION OF COURSE 2 TO NCTM STANDARDS

Course 2 Units / NCTM Grades 9–12 Content Standards	Algebra	Functions	Geometry from a Synthetic Viewpoint	Geometry from an Algebraic Viewpoint	Trigonometry	Statistics	Probability	Discrete Mathematics	Conceptual Underpinnings of Calculus	Mathematical Structure
Matrix Models	✔	✔	+			+		✔		✔
Patterns of Location, Shape, and Size	✔	✔	+	✔				✔		+
Patterns of Association	✔	✔		+		✔		+		
Power Models	✔	✔	+			✔		+	✔	✔
Network Optimization	✔		+					✔		
Geometric Form and Its Function	✔	✔	✔	+	✔				+	
Patterns in Chance	+	+	+			✔	✔	+	+	
Capstone—Forests, the Environment, and Mathematics	✔	✔	✔	✔	✔	✔	✔	✔	✔	

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OVERVIEW OF COURSE 3

Unit 1 **Multiple-Variable Models** develops student ability to construct and reason with linked quantitative variables and relations involving several variables and several constraints.

Topics include: Formulas that relate several variables by a single equation such as the Law of Sines and the Law of Cosines; systems of equations with several dependent variables and/or constraints; patterns of change in one or more variables in response to changes in the others; solution of systems of equations and inequalities; and linear programming.

Unit 2 **Modeling Public Opinion** develops student understanding of how public opinion can be measured. The situations analyzed include elections (where there are more than two choices) and sample surveys, including political polling.

Topics include: Preferential voting, vote analysis methods, Arrow's theorem, fairness in social decision making, surveys, sampling, sampling distributions, relationship between a sample and a population, confidence intervals, margin of error, and critical analysis of elections and surveys.

Unit 3 **Symbol Sense and Algebraic Reasoning** develops student ability to represent and draw inferences about algebraic relations and functions using symbolic expressions and manipulations.

Topics include: Formalization of function concept, notation, domain, and range; use of polynomial, exponential, and rational expressions to model relations among quantitative variables, field properties of real numbers and their application to expression of algebraic relations in equivalent forms and to solution of equations and inequalities by methods including factoring and the quadratic formula; and algebraic proof.

Unit 4 **Shapes and Geometric Reasoning** introduces students to formal reasoning and deduction in geometric settings.

Topics include: Inductive and deductive reasoning, counterexamples, the role of assumptions in proof, conclusions concerning supplementary and vertical angles and the angles formed by parallel lines and transversals, conditions insuring similarity and congruence of triangles and their application to quadrilaterals and other shapes, and necessary and sufficient conditions for parallelograms.

Unit 5 **Patterns in Variation** extends student understanding of the measurement of variation, develops student ability to use the normal distribution as a model of variation, and introduces students to the probability and statistical inference involved in the control charts used in industry for statistical process control.

Topics include: Standard deviation and its properties, normal distribution and its relation to standard deviation, statistical process control, control charts, control limits, mutually exclusive events, and the addition rule of probability.

Unit 6 **Families of Functions** reviews and extends student ability to recognize different function patterns in numerical and graphical data and to interpret and construct appropriate symbolic representations modeling those data patterns.

CONTINUED

Topics include: Review of linear, polynomial, exponential, rational, and periodic functions (including effects of parameters on numeric and graphic patterns); and construction of function rules for function tables and graphs that are transformations of basic types (translation, reflection, stretch).

Unit 7 **Discrete Models of Change** extends student ability to represent, analyze, and solve problems in situations involving sequential and recursive change.

Topics include: Iteration and recursion as tools to model and analyze sequential change in real-world contexts; arithmetic, geometric, and other sequences; arithmetic and geometric series; finite differences; linear and nonlinear recurrence relations; and function iteration, including graphical iteration and fixed points.

CAPSTONE **Looking Back at Course 3: Making the Best of It: Optimal Forms and Strategies** is a thematic, two-week, project-oriented activity that enables students to pull together and apply the important mathematical concepts and methods developed throughout the course.

The *Contemporary Mathematics in Context* curriculum and the instructional and assessment practices it promotes address the focal points of the National Council of Teachers of Mathematics' *Curriculum Standards* for grades 9–12. By design, the **process standards** for Problem Solving, Reasoning, Communication, and Connections are an integral part of each lesson of every unit in the curriculum.

The chart below correlates Course 3 units with the **content standards** for grades 9–12 in terms of primary emphases (✔) and connections (+).

CORRELATION OF COURSE 3 TO NCTM STANDARDS

Course 3 Units	NCTM Grades 9–12 Content Standards	Algebra	Functions	Geometry from a Synthetic Viewpoint	Geometry from an Algebraic Viewpoint	Trigonometry	Statistics	Probability	Discrete Mathematics	Conceptual Underpinnings of Calculus	Mathematical Structure
Multiple-Variable Models		✔	✔	+	+	✔			✔	+	+
Modeling Public Opinion				+			✔	✔	✔		
Symbol Sense and Algebraic Reasoning		✔	✔	✔	+	+	+			✔	✔
Shapes and Geometric Reasoning		✔	+	✔	✔	✔	+		+		✔
Patterns in Variation		+	+				✔	✔	+		
Families of Functions		✔	✔	+	+	✔	+	+		✔	
Discrete Models of Change		✔	✔	+				+	✔	+	✔
Capstone—Making the Best of It: Optimal Forms and Strategies		✔	✔	✔	✔	✔	✔	✔	✔	+	

INSTRUCTIONAL MODEL

The manner in which students encounter mathematical ideas can be as important as the mathematics to which they are introduced. Therefore, *Contemporary Mathematics in Context* is designed around specific cycles of instructional activities intended primarily for small-group work in the classroom and for individual work outside of the classroom.

In Class

Each multiday lesson uses a four-phase cycle of classroom activities—*Launch, Explore, Share and Summarize,* and *Apply*—designed to actively engage students in investigating and making sense of problem situations, in constructing important mathematical concepts and methods, and in communicating their thinking and the results of their efforts. The summary below describes these phases of classroom work.

IN-CLASS ACTIVITIES

Launch Lessons begin with a full-class discussion of a problem situation and of related questions to **think about**. This discussion sets the context for the student work to follow and helps to generate student interest; it also provides an opportunity for the teacher to assess student knowledge and to clarify directions for the group activities. *Teacher is director and moderator.*

Explore Classroom activity then shifts to **investigating** focused problems and questions related to the launching situation by gathering data, looking for patterns, constructing models and meanings, and making and verifying conjectures. As students work collaboratively in small groups, the teacher circulates from group to group providing guidance and support, clarifying or asking questions, giving hints, providing encouragement, and drawing group members into the discussion to help groups work more collaboratively. The unit materials and related questions posed by students drive the learning. *Teacher is facilitator.*

Share and Summarize A full-class discussion (referred to as a "Checkpoint") of concepts and methods developed by different small groups then provides an opportunity to **share** progress and thinking. This discussion leads to a class **summary** of important ideas or to further exploration of a topic if competing perspectives remain. Varying points of view and differing conclusions that can be justified should be encouraged. *Teacher is moderator.*

Apply Finally, students are given a task to complete on their own to **reinforce** their initial understanding of a concept or method. The teacher circulates in the room assessing levels of understanding. *Teacher is intellectual coach.*

Most classroom activities are designed to be completed by students working together collaboratively in heterogeneous groupings of two to four. Because of the importance of collaborative learning in this curriculum, the section "Active Learning and Collaborative Work" beginning on page 21 provides detailed methods and procedures to assist teachers in facilitating collaborative learning in the classroom.

Out of Class

In addition to the classroom investigations, *Contemporary Mathematics in Context* provides sets of MORE tasks, which are designed to engage students in ***Modeling*** with, ***Organizing***, ***Reflecting*** on, and ***Extending*** their mathematical knowledge. These tasks, which are central to the learning goals of each lesson, are intended primarily for individual work outside of class. The summary below describes the types of tasks in a typical MORE set.

OUT-OF-CLASS ACTIVITIES

Modeling Modeling tasks are related to, or provide new contexts to which students can apply, the ideas and methods that they have developed in the lesson.

Organizing Organizing tasks offer opportunities for integrating the formal mathematics underlying the mathematical models developed in the lesson and for making connections with other strands.

Reflecting Reflecting tasks encourage thinking about thinking itself, about mathematical meanings, about processes, and promote self-monitoring and evaluation of understanding.

Extending Extending tasks permit further, deeper, or more formal study of the topics under investigation.

Several MORE tasks are provided for each lesson. Selection of MORE tasks for use with a given class should be based on student performance and the availability of time and technology. Also, students should exercise some choice of tasks to pursue, and at times they should be given the opportunity to pose their own problems and questions to investigate.

Assessment

Assessing what students know and are able to do is an integral part of *Contemporary Mathematics in Context*, and there are opportunities for assessment in each phase of the instructional model. Initially, as students pursue the investigations that make up the curriculum, the teacher is able to informally assess student performance in terms of process, content, and disposition or other factors. Then, at the end of each investigation, the "Checkpoint" and class discussion provide an opportunity for teachers to assess the levels of understanding that the various groups of students have reached as they share and summarize their findings. Finally, "On Your Own" problem situations and the MORE sets provide additional means for assessing the level of understanding of each individual student.

The methods for conducting assessments include fundamental classroom techniques of observation, questioning, and listening as well as specific assessment materials. Because the student materials consist largely of questions rather than statements of information, the teacher will have ample opportunity to observe and assess how students think about and apply mathematics. The *Assessment Resources* contains end-of-lesson quizzes, end-of-unit exams (both in-class and take-home), and projects. In addition, many teachers have their students write about the investigations and discussions in personal journals, which can provide valuable insight into various aspects of student understanding and attitude. Some teachers have their students assemble portfolios of their work, and these may be used in some longer-term assessments.

A much more detailed description of the *Contemporary Mathematics in Context* assessment program can be found beginning on page 35.

Implementing the Curriculum

PLANNING FOR INSTRUCTION

The *Contemporary Mathematics in Context* curriculum is not only changing what mathematics all students have the opportunity to learn, but also changing how that learning occurs and is assessed. Active learning is most effective when accompanied with active teaching. Just as the student texts are designed to support students who are actively engaged in doing mathematics, the teacher's resource materials are designed to support teachers who are actively involved in planning for instruction; in observing, listening to, questioning, facilitating student work, and orchestrating classroom discussion; and in managing the classroom.

The *Teacher's Guides* provide suggestions, based on the experiences of field-test teachers, for implementing this exciting new curriculum in your classroom. You probably will find several new ideas that can be overwhelming. The developers highly recommend that teachers who are teaching *Contemporary Mathematics in Context* for the first time do so at least in pairs who share a common planning period.

Each of the items listed below are included in the *Teacher's Guides* for each unit and the capstone.

- Unit overview
- Objectives, suggested time line, and materials needed
- Instructional notes and suggestions
- Suggested assignments for each MORE set
- Solutions for Investigations and MORE tasks
- Unit summary and a look ahead

Teaching Resources include blackline masters for creating transparencies and handouts. *Assessment Resources* include quizzes for individual lessons, end-of-unit exams, and take-home assessment activities. Special calculator software has been developed to support students' investigations and modeling applications in

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each of the four strands. The software for the TI-82, TI-83, and TI-92 graphics calculators is available on disk for downloading from Macintosh and DOS- or Windows-based (PC) computers.

Each unit of *Contemporary Mathematics in Context* includes either content which may be new to many teachers or new approaches to familiar content. Thus a first step toward planning the teaching of a unit is to review the scope and sequence of the unit itself. This review provides an overall feel for the goals of the unit and how it holds together. Working through the student investigations, if possible with a colleague during common planning periods, provides help in thinking about and understanding mathematical ideas that may be unfamiliar.

In the *Teacher's Guides* you will find teaching notes for each lesson, including instructional suggestions and sample student responses to investigations and MORE sets. Thinking about the range of possible responses and solutions to problems in a lesson proves to be very helpful in facilitating student work.

A summary chart at the beginning of the notes for each unit gives information about materials needed, time guidelines, and recommended assignments from the MORE sets. The Unit 2 Planning Guide for Course 1 is shown on the next page.

You will notice in the Planning Guide that the suggested time for the third lesson is approximately five 50-minute instructional periods. This may vary depending on the background and make-up of the class. Remember: *developing deep understanding is more important than just "completing activities."*

Since acquiring the physical materials requires advance planning, specific materials for each lesson are listed in the Planning Guide. Although it is not stated in the Planning Guide, it is assumed that students have access to graphics calculators at all times for in-class work.

The developers recommend that the homework (MORE) assignment *not* be held off until the end of the lesson or the investigation just preceding the MORE set. Some teachers choose to post the MORE assignment at the beginning of a lesson along with the due date—usually a day or two following planned completion of the lesson. Other teachers prefer to assign particular Modeling and Reflecting tasks at appropriate times during the course of the multiday lesson and then assign the remaining tasks toward the end of the lesson. Note that all recommended assignments include provision for student choice of some tasks. This is but one of many ways in which this curriculum is designed to accommodate and support differences in students' interests and performance levels.

It is strongly recommended that student solutions to Organizing tasks be discussed in class. These tasks help students organize and formalize the mathematics developed in context and connect it to other mathematics they have studied. Structuring the underlying mathematics and building connections is best accomplished by comparing and discussing student work and synthesizing key ideas within the classroom.

Unit 2 Planning Guide

Lesson Objectives	MORE Assignments	Suggested Pacing	Materials
Lesson 1 *Related Variables* • To identify key variables in the situation to be modeled • To collect data that will suggest the pattern relating those variables • To make a table and graph the data to look for patterns • To make predictions that go beyond the data (extrapolations and interpolations)	**page 102** **Modeling:** 2 and choice of one* **Organizing:** 2, 4 **Reflecting:** 3 **Extending:** 1	2 days	• Each group: rubber bands, fishing weights, graph paper • Teaching Resources 46–48 • Assessment Resources 39–44
Lesson 2 *What's Next?* • To focus students' attention on patterns of change in variables • To explore iterative or recursive change • To examine equations relating *NOW* and *NEXT* • To introduce and utilize the iteration capabilities of a graphing calculator or computer software	**after page 113** Students can begin Modeling Tasks 1 or 2 from p. 116 **page 116** **Modeling:** 1 or 4, 2 or 3* **Organizing:** 2, 4 **Reflecting:** 1 **Extending:** 1	4 days	• Teaching Resources 49–51 • Assessment Resources 45–50
Lesson 3 *Variables and Rules* • To develop students' ability to summarize patterns relating variables using rules • To utilize rules along with a graphing calculator or computer software • To illustrate the power of a rule and its capabilities when used with a graphing calculator or computer software • To explore difficult questions relating variables by using rules and technology	**after page 125** Students can begin Modeling Tasks 1 and 4; Organizing Task 1; or Extending Tasks 1 or 2 from p. 132 **page 132** **Modeling:** 1 and choice of one* **Organizing:** 1 or 2, 4* **Reflecting:** 1 **Extending:** 2	5 days	• Advertisements of TVs, CD players, *etc.*, with prices • Teaching Resources 52–58 • Assessment Resources 51–56
Lesson 4 *Linear and Nonlinear Patterns* • To begin to understand and develop an appreciation for the power of technology-generated tables and graphs and the essential role of algebraic rules in providing directions to a calculator or computer software • To recognize the connection between two-variable relationships and their graphs	**after page 144** Students can begin Modeling Tasks 3–5; Reflecting Task 4; or Extending Task 3 from p. 147 **page 147** **Modeling:** 3 and choice of one* **Organizing:** 1, 3 **Reflecting:** 3 and choice of one* **Extending:** 3	4 days	• Optional: Tennis ball or softball • Teaching Resources 59–65 • Assessment Resources 57–62
Lesson 5 *Looking Back* • To review the major objectives of the unit		3 days (including testing)	• Graph paper • Teaching Resource 66 • Assessment Resources 63–78

* *When choice is indicated, it is important to leave the choice to the students.*

Note: *It is best if Organizing tasks are discussed as a whole class after they have been assigned as homework.*

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ORCHESTRATING LESSONS

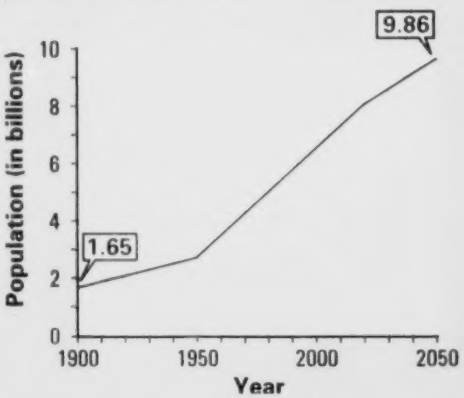
The *Contemporary Mathematics in Context* materials are designed to engage students actively in a four-phase cycle of classroom activities. (See "Instructional Model" on page 12.) The activities often require both students and teachers to assume roles quite different than those in more traditional mathematics classrooms. Becoming accustomed to these new roles usually takes time, perhaps a semester or more, but field-test teachers report that the time and effort required are well worth it in terms of student learning and professional fulfillment. Although realistic problem solving and investigative work by students is the heart of the curriculum, how teachers orchestrate the launching of that activity and the sharing and summarizing of results is critical to successful implementation. Lessons should be introduced by asking students to think about a situation such as the one below, which is used to launch Lesson 2 of the "Patterns of Change" unit in Course 1.

What's Next?

The United States government—as well as local, state, and other national governments around the world—does lots of counting. Government agencies make hundreds of census counts every year. They keep track of jobs and unemployment, automobiles, accidents, animals, illnesses, forests, and farmlands. Every ten years, the U.S. Census Bureau counts every American citizen.

By the mid-1990s, the world's population was estimated to be about 5.6 billion. Some projections estimate it will almost double by 2050.

Explosive Growth



Source: *World Population Prospects*, The 1994 Revision. New York: United Nations, 1995.

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Think About This Situation

The population of our world changes rapidly—in ten years it is expected to grow by over one billion people.

- **a** What are some of the major factors that might influence population change in a country?
- **b** Why doesn't the U. S. Census Bureau make a complete count of the population every five years or every year?
- **c** Why would it be important to know year-to-year population changes, and how you could estimate those changes without a full census survey?

Students enter the classroom with markedly different backgrounds, experience, and knowledge. These differences can be viewed as assets. Engaging the class in a free-flowing give-and-take discussion of how they think about the launch situations serves to connect lessons with the informal understandings of data, shape, change, and chance that students bring to the classroom. Try to maximize the participation of students in these discussions by emphasizing that their ideas and possible approaches are valued and important and that definitive answers are not expected at this time.

Once launched, a lesson may involve students working together collaboratively in small groups for a period of days punctuated occasionally by brief, whole-class discussion of questions students have raised. In this setting, the lesson becomes driven primarily by the instructional materials themselves. Rather than orchestrating class discussion, the teacher shifts to circulating among the groups and observing, listening, and interacting with students by asking guiding or probing questions. These small-group investigations lead to (re)invention of important mathematics that makes sense to students. Sharing, and agreeing as a class, on the mathematical ideas that groups are developing is the purpose of the "Checkpoints" in the instructional materials.

Students first should decide on responses to "Checkpoint" questions as a group. The sample "Checkpoint" shown on the next page is the first of two in Lesson 2 of "Patterns of Change".

Class discussions at "Checkpoints" are orchestrated somewhat differently than during the launch of a lesson. At this stage, mathematical ideas and methods still may be under development and may vary for individual groups. So class discussion should involve groups comparing their methods and results, analyzing their work, and arriving at conclusions agreed upon by the class.

Checkpoint

In population studies of Brazil and the United States, you made estimates for several years, based on growth trends from the past.

ⓐ What calculations are needed to estimate population growth from one year to the next in the two different countries?

ⓑ Using the word *NOW* to stand for the population of the United States in any year, write an expression that shows how to calculate the population in the *NEXT* year.

Be prepared to compare your expression relating* NOW *and* NEXT *with those of other groups.

For each part of the "Checkpoint," different groups should be asked to share their responses and thinking before proceeding to the next part. Facilitate resolution of any differences as a step toward building agreement on the class's mathematics.

The investigations deepen students' understanding of mathematical ideas and extend their mathematical language in contexts. Occasionally there is a need to introduce conventional or more technical terminology and symbolism. This occurs in the student text immediately following a "Checkpoint" and before the corresponding "On Your Own" task. These connections should be introduced by the teacher as a natural way of closing the class discussion summarizing the "Checkpoint."

On Your Own

a. Write an expression using *NOW* and *NEXT* that shows how to use the population of Brazil in one year to calculate the population in the next year.

b. If the birth rate in the United States increased to 2.6%, like Brazil, how would that change the population increase from one year to the next for the U.S.? Estimate the 1991 population using the larger birth rate.

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Managing Classroom Activities

Teachers who have been teaching the *Contemporary Mathematics in Context* curriculum have found a variety of ways to manage classroom activities. Initially, the two biggest challenges are facilitating cooperative group work and organizing, monitoring, and assessing student work. The following section on collaborative work offers a number of suggestions for meeting the first challenge. The second challenge is the focus of the sections on assessment (pages 35–46 of this guide) and tips on student work and writing (pages 47–48).

ACTIVE LEARNING AND COLLABORATIVE WORK

The Contemporary Mathematics in Context curriculum materials are designed to promote active, collaborative learning and group work. Creating a classroom atmosphere for effective collaborative learning requires the following: an understanding of the basic philosophy behind cooperative learning; familiarity with the factors in the composition and selection of groups; and practice in classroom techniques for managing cooperative learning. The following is a summary of, and a ready reference to, these aspects of the cooperative learning model.

Philosophy Behind Collaborative Learning

Contemporary Mathematics in Context deliberately incorporates small-group collaborative learning for two reasons. First, cooperative learning is the most effective method for engaging *all* the students in a class in the learning process. Second, practice in cooperative learning in the classroom is practice for real life: students develop and exercise the same skills in the classroom that they need in their lives at home, in the community, and in the workplace.

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VALUE OF INDIVIDUALS

Perhaps the most fundamental belief underlying the use of cooperative learning is that every student is viewed as a valuable resource and contributor. In other words, every student participates in group work and is given the opportunity and time to voice ideas and opinions. Implementing this concept is not easy. It does not happen automatically. In order to set a tone that will promote respect for individuals and their contributions, classroom rules should be established and agreed upon by the learning community. Students should be included in the process of formulating the rules. Helpful rules might include the following:

- Refer to people by the names that they prefer (provided, of course, that the names are appropriate and not offensive); there will be no name calling.
- Speak respectfully; for example, do not say "Shut up."
- Acknowledge that everyone's ideas have value; listen to others, even if you disagree.
- Direct any criticism at ideas, not at people.
- Be willing to change your mind on the basis of logic.
- Include everyone. Do not leave anyone out on purpose or by chance. If you are placed in groups with others who you don't like, do not exclude them from the group discussion or from making decisions.
- Collaborate rather than compete. Group work is for cooperation, and one group is not in competition for answers or time with any other group in the room.
- As a last resort, agree to disagree.

The teacher should initiate a discussion of group rules and then post them in the classroom. The teacher should model all of the rules correctly to show that "we" begins with "me." Those who do not adhere to the rules must accept the consequences in accordance with classroom or school disciplinary procedures.

IMPORTANCE OF SOCIAL CONNECTIONS

Even in classrooms in which the rules for showing respect have been clearly established, experience has shown that students still cannot talk with one another about mathematics (or social studies, or literature, or any other subject) if they do not first have positive social connections.

One way to develop this kind of common base is through team-building activities. These short activities (five to ten minutes) may be used at the beginning of the year to help students get acquainted with the whole class and may be used during the year whenever new groups are formed to help groupmates know one another better.

In one such activity,[1] called "Whip," the members of each group give quick answers to a statement given by the teacher, such as:

[1] Other activities are available in *The Guidebook for Cooperative Learning*, Dishon and Wilson O'Leary, (Learning Publications, Holmes Beach, FL, 1994) and in *Cooperative Learning in Mathematics: A Handbook for Teachers*, edited by Neil Davidson, (Addison Wesley Publishing Company, New York, NY, 1990).

"My favorite pastime is _____."

"My favorite vacation memory is _____."

"Something new and positive in my life is _____."

"A hobby or sport I like is _____."

These kinds of activities help students learn new and positive things about classmates with whom they may have attended classes for years, but have not known in depth. The time taken for these quick team-builders pays off later in helping students feel comfortable enough to work with the members of their group.

NEED FOR TEACHING SOCIAL SKILLS

Experience also has shown that social skills are critical to the successful functioning of any small group. Because there is no guarantee that students of any particular age will have the social skills necessary for effective group work, it often is necessary to teach these skills to build a collaborative learning environment.

These social skills are specific skills, not general goals. Examples of specific social skills that the teacher can teach in the classroom include responding to ideas respectfully, keeping track of time, disagreeing in an agreeable way, involving everyone, and following directions. Though goals such as cooperating and listening are important, they are too general to teach and practice.

One method of teaching social skills is to begin by selecting a specific skill and then having the class brainstorm to develop a script for practicing that skill. Next, the students practice that skill during their group work. Finally, in what is called processing, the students discuss within their groups how well they performed the assigned social skill. Effective teaching of social skills requires practicing and processing; merely describing a specific social skill is not enough. Actual practice and processing are necessary for students really to learn the skill and to increase the use of appropriate behaviors during group work and other times during class.

Composition, Selection, and Management of Groups

One of the premises of cooperative learning is that by developing the appropriate skills through practice, anyone in the class can learn to work in a group with anyone else. Learning to work in groups is a continuous process, however, and the process can be helped by decisions that the teacher makes with regard to the size, composition, method of selection, student reaction to, and duration of groups. Attention to dealing effectively with student absences also is important.

GROUP SIZE

Any group must have at least two students, and probably the largest manageable groups have five. Groups of different sizes may be appropriate for different purposes or for different reasons. There are several factors to consider in determining group size.

One factor is time available. The larger the group, the more time it takes to include everyone's ideas and to reach consensus. The smaller the group, the less time it takes to complete the work and include everyone. Thus the amount of time allotted for a particular group assignment may help determine the best group size.

A related factor in determining group size is the level of social skills exhibited by members of the group. Larger groups require higher levels of patience, listening skills, and ability to accept different perspectives, while smaller groups can succeed with fewer social skills and lower levels of skills in the group.

Another factor is the willingness of students to engage in group activities. If the class contains many students who tend to hang back and avoid involvement, the best group size is two. It is hard to be left out in a group of two, and students must be more accountable when working in pairs.

Finally, if the group task involves problem solving or the need to generate a variety of ideas, then larger groups may be best. Students are ready for groups of four or even five if they are willing and able to take the time—and if they have the skills—to include everyone. One transitional alternative is to have students work in pairs initially and then have each pair share with another pair. If grouping is by 3s and there are extra persons, then one or more groups are increased to 4.

HETEROGENEITY

The strongest groups contain students who are different from one another in gender, ethnicity, skills and abilities, personalities, socio-economic background, previous school experiences, and so forth. When a heterogeneous group of students work together, they bring different ideas, experiences, and points of view. These differences strengthen the group.

METHODS OF SELECTION

Given the above principles for forming groups, there are at least three methods of actually determining which students will be in which groups.

One way to form groups is for the teacher to determine the groupings: the teacher picks individual students to work with other individual students, based on considerations of group size and heterogeneity. The teacher, however, should not attempt to create the perfect group by trying to blend all of the criteria at one time; rather, the teacher should base the groupings on the tasks at hand and on the characteristics of the class. For example, if the small groups require reading skills, individuals should be grouped so that there is a range of reading skills available in each group; or, if members of a class of freshmen tend to sit and talk only with students from their own middle school, then the groupings should mix students from different feeder schools.

Another method for determining groups is random selection from some kind of lottery or chance drawing. While the teacher will want to use a variety of methods during the year, one method is to simply write each student's name on a slip of paper, put the papers in a container, and then draw out the number of

names corresponding to the size of the group that the teacher has determined for the particular activity.

When a random method is used, both teacher and students must be ready for new and possibly untried combinations of students. Students often like random selection best because it seems fair, while teacher selection appears to be "fixed."

A third method that generally is not recommended is letting students select the groups themselves. When students pick their own partners, they tend to pick their friends, people who are like themselves, not different. Some students may not be picked at all. Student selection therefore can violate the basic premises of cooperative learning.

Student selection, however, may be appropriate within limits. For example, the teacher might say, "You may pick your groups today. Find three other students (total of four), include males and females, and make sure that you are in a group with students who you do not sit with at lunch or hang out with after school. No one may be left out." If students can follow these guidelines (or others that the teacher chooses), then student-selected groups may be used on occasion.

STUDENT REACTION TO GROUP COMPOSITION

No matter which method is used to select the members of each group, the students will have mixed reactions to the groupings. The teacher should start very early to set limits on the acceptable reactions by discussing with the students what they may and may not say when they find out who is in their group. Some possible guidelines include:

- Groups will be changed throughout the semester and the year, but they will not be changed until any present problems are resolved.
- You must treat groupmates with respect.

DURATION OF GROUPS

Because different groups will serve different purposes, the length of time that a group stays together will vary. For example, two-person groups of students who sit next to one another might check homework at the beginning of the class period for three to five minutes. Two pairs might combine to create a four-person group for investigations. Work groups for projects or investigations might remain the same for each unit or even for a nine-week grading period. The goal is to have the students work with many different combinations of classmates during the school year.

DEALING WITH STUDENT ABSENCES

"My partner isn't here today? Who do I meet with?" "He has the group paper, but he's gone today!" Whatever the composition and duration of the groups, absentees can pose problems but they are manageable ones. (Some studies have

even indicated that students increase their attendance in classes that use cooperative learning.) When a missing group member has materials the group needs, it is difficult for the group to continue its work. When students are absent, they miss the content and the practice provided by the group work. Here are some suggestions for dealing with problems related to student absences.

- For long-term groups (two to nine weeks), use groups of four students. With groups of four, it is very likely that there will be at least two or three students in the group on a given day, so the group is still a group, and the teacher can avoid the awkward and time-consuming task of moving a lone person to a new group.
- Do not carry over completed activities into additional class periods to provide catch-up time for students who were absent.
- For short-term activities that cover two or more class periods, decide the placement of a returning student on a case-by-case basis. If a student has missed one day of a three-day investigation, the student may be placed in a group. If a student has missed two days of a three-day investigation, the student might be asked to do a shortened, individual investigation addressing the same content.
- When a student returning to class does not have an assigned group, place the student in a group. Pick the best group for the student—*i.e.*, pick a group that is friendly, patient, and likely to include the student as a working member.
- Never let an ongoing group's work (project, research notes, etc.) leave the room. For any continuing activity, have group folders or a group basket ready. When class is over, all group papers go into the folder or basket. The next day, the group has its work, regardless of student attendance.
- Encourage each group to be responsible for helping absent members make-up missed work by giving them an update by phone before class or immediately upon their return to class.

Classroom Techniques for Effective Collaborative Learning

While productive and congenial collaborative learning depends to a significant degree upon student attitudes and skills and upon the composition of groups, much of the success of group learning depends upon the teacher's management of the classroom. Key areas for insuring effective group work include (1) giving clear directions, (2) establishing a materials center, (3) setting up the work area for face-to-face interaction, (4) establishing classroom routines, (5) using a variety of techniques to develop positive interdependence, (6) establishing individual accountability, and (7) handling conflict effectively.

GIVING CLEAR DIRECTIONS

Explicit, well-organized directions are essential for effective group work. Because of the variety of reading levels and differences in learning styles that may exist in groups, it is helpful if the teacher displays the directions using an overhead projector. Reading those directions aloud and checking for understand-

ing before students move into groups helps auditory learners. Keeping the directions displayed during work time helps visual learners. If asked a question that is answered in the directions, the teacher responds by simply pointing to the display. Some experts suggest that the teacher should always stand in the same spot in the room when giving directions.

Clear directions for activities in *Contemporary Mathematics in Context* are often provided in the student text; however, if the teacher creates the directions for activities, care should be taken that those directions are also clear. Time spent in planning pays off later.

ESTABLISHING A MATERIALS CENTER

Another way to help maximize efficiency in a collaborative learning environment is to establish a materials center, a single location where designated students can get all supplies for their groups and keep their group projects. The teacher should place the supplies there before class or at the start of work time. Students should get in the routine of getting all materials from, and returning them to, the materials center. It is frustrating to all if students can't find, or waste time looking for, needed items such as calculators, rulers, or handouts. A materials center saves time and encourages both the teacher and the students to be organized.

SETTING UP FOR FACE-TO-FACE INTERACTION

The physical location of students in the room affects how they respond to the content of the lesson, to the teacher, and to each other. For group work, students need to be "eye-to-eye" and "knee-to-knee." In other words, students in the same group should face their groupmates and face away from the teacher and other groups. A minimum of furniture is best. Less table surface is needed when students share materials than when all students have their own materials. (See the discussion on "limited" and "jigsawed" materials on page 29–30 of this guide.) With shared materials, three or four chairs can be clustered around a single desk or at the end of a rectangular table. In any group, students should be close together so that everyone can see easily and can hear without talking loudly.

ESTABLISHING CLASSROOM ROUTINES

Establishing routines for getting into groups, moving furniture, and having the class become quiet all help to facilitate cooperative learning. The Dishon/Wilson O'Leary model recommends the use of the "3 Rs" to teach a routine: *reveal*, *rehearse*, and *reinforce* (Dishon and Wilson O'Leary, 1994). A "Quiet Signal" is one important routine for a cooperative classroom.

1. *Reveal:* Name the routine, explain why it is needed, tell how it works, and demonstrate the behavior.

   "The routine you will learn today is called the Quiet Signal. It is necessary to have a Quiet Signal because you will be busy working in groups and at some point I will need your attention. I would like to do this quietly so that you can quickly finish your sentence and give me your

attention. I will simply raise my hand. If you are not talking and see my hand, continue listening to whomever is speaking and raise your hand. If you are speaking, finish your sentence and then raise your hand to acknowledge the signal." At this point, the teacher raises a hand.

2. *Rehearse:* Students practice the procedure while the teacher observes. The students should practice several times under different conditions.

   "Now I would like to have you practice this routine. In a moment, I want you to turn to your neighbor and tell about something you like to do on the weekend. Then listen while your neighbor does the same. You will have two minutes." The teacher moves quietly around the room listening while students talk. At the end of two minutes, the teacher stands quietly at the front of the room and raises one hand. Students' hands go up slowly at first and then more quickly as more students see the signal. "Thank you. That worked quite well. Remember to finish your sentence before you raise your hand. This time let's practice with several people walking around the room, some people talking quietly, and the rest working at your desks." This time the teacher lets students' movement continue for 30 seconds or so and then raises one hand. "I noticed people seeing my hand and raising one of theirs even when they were out of their seats. That was just fine!"

3. *Reinforce:* It is important to practice, give feedback, review, and reteach if necessary until students use the routine consistently in appropriate ways. Also, on some occasions after using the routine, be sure to ask students for their perception of how it is working.

   "I want to ask you about how the Quiet Signal is working. Did the person who was speaking when the Quiet Signal was noticed finish the sentence?" "What can you do so that the person talking finishes the sentence and doesn't start another one?" Students offer possibilities. The discussion continues and another brief practice session is conducted.

One caution: be sure that you are using the routine as it was taught. Teachers who have problems with the Quiet Signal often are talking while they have their hand up or are walking around reminding people to be quiet. Teach the routine by standing quietly, not looking directly at any student, and then *wait* until it is quiet. Taking the time to teach the routine will be beneficial to you, your students, and successful cooperative group work.

DEVELOPING POSITIVE INTERDEPENDENCE

Positive interdependence, the extrinsic conditions that motivate students to work together, is at the heart of collaborative learning. There are various types of tools for building positive interdependence:

- focus on the group for directions, questions, goals, and products;
- share materials;
- share roles.

At least one of these tools must be used during any cooperative learning lesson, although more can be included.

The most obvious tools for ensuring that students work together in their groups have to do with focusing on group processes. For example, the procedures described above for giving directions clearly focus on the group and reinforce the principle of group structure.

A logical corollary to giving group directions is the practice of answering only group questions. When students are working in a group, the teacher should establish the general practice of answering only those questions that have been decided upon by the group. If a student has a question about the group's work, the student should first ask the group. If no one has a response that helps, or if there are various responses and the group cannot reach agreement on any of them, this group of students may ask the teacher. When the group has a question, all group members must raise a hand, thus signaling to the teacher that everyone agrees that help from the teacher is needed. It should be understood by all that, when the teacher arrives at the group, the teacher may call on *anyone* in the group, not just the person who asked the original question. If the person called on does not know the question, the teacher may say, "I will come back when you have a group question." If the person called on knows the question, the teacher may ask another question, give a hint to help the group determine an appropriate answer, refer them to another group, or answer the question.

The belief behind the practice of the group question is that the group is a student's main resource. It is important for students to realize that they can think and solve problems within their groups. The teacher's job is to facilitate group work, not to participate as a member of any group.

Before beginning group work, it would be helpful for students to understand the reasons for responding only to group questions. Students may resist this new way of working, and teachers may have difficulty using the routine regularly. In the past, teachers may have rushed to answer every question, but this change will help students think for themselves.

Establishing a group goal is another way to help students achieve positive interdependence within their groups. Such a goal might be to work through an investigation, or to prepare for a class discussion at a "Checkpoint." During their group work, students may make and confirm conjectures, discuss ideas, decide on strategies, and so on.

Still another way to increase the likelihood that students will think, talk, and work together in a group is to have the group create a single group product. The one product, which must be specifically described in the directions for the activity, is worked on by everyone in the group: everyone must give ideas or opinions; everyone must write, or draw, or press the calculator keys when agreed to by the group. This product is not created in an assembly line, like an add-a-line poem; rather, it is shared, but not divided, equally among the group members, with group approval.

Another tool for developing positive interdependence is sharing materials in one of two ways. With "limited" materials, the group is limited to one set of materials. If the activity requires a ruler, marker, scale, and calculator, there is only one of each item in the group. These materials then rotate among the

members of the group, with each student taking a turn measuring, writing, weighing, or calculating, at the direction of the group.

With "jigsawed" materials, the different items needed for the activity are distributed among the group members. In other words, everyone in the group has something that the other group members need. If measurements are required, each person has a different tool (ruler, scale, calculator), and all tools must be used to complete the assignment. If colored markers are needed for an activity, each student in the group has a different colored marker, and the group product requires use of all colors. Or each student may have a different piece of information, and the group needs all the pieces of information to respond to the problem at hand.

Sharing materials in these ways helps ensure that the quiet, shy, or lazy student is included in the activity, and the sharing also helps reduce the influence of dominating students who want to use only their own ideas. Because individual students who have their own materials have less reason to interact with other group members, they are more likely simply to socialize in their talking or not talk at all. Sharing materials thus encourages students to engage in conversation about the work, to explain, predict, negotiate, and reach consensus.

Sharing roles is also a type of tool to develop interdependence within groups, and there are several approaches to the assignment of roles. In what Dishon and Wilson O'Leary call "specifically assigned rotated roles," each student in the group has a particular function for a particular group session. This method of assignment can be seen in the references in the student texts to the roles of Reader, Recorder, Quality Controller, and Coordinator. Depending upon the circumstances and the task at hand, of course, there can be different combinations of roles within implementation of investigations. For example, when students work in groups of three, the roles of Quality Controller and Coordinator are often combined. With this type of role sharing, the teacher may assign roles to every student most of the time, and the teacher may rotate the job assignments before the beginning of each new investigation or each new lesson.

In another type of role sharing, everyone is responsible for taking a turn at each job during group work time. If there is measuring to be done, each student takes a turn being the measurer; if there is writing to be done, each student takes a turn at being the writer; when it is time to report on group work, each student does part of the reporting. All group members are thus responsible for contributing ideas and information, as well as for checking the accuracy of the group's work.

Still another way to deal with roles is not to label or assign them at all. Instead, after the group's work is completed, the teacher asks students to reflect on the tasks that had to be done and how these tasks were handled.

ESTABLISHING INDIVIDUAL ACCOUNTABILITY

While the use of the techniques described above for development of positive interdependence is a major theme of group work, establishing individual accountability is also a very important component of collaborative learning.

One way to start developing accountability is to require individual student signatures on group products. At the end of work time, each student signs the group product with full name, not scrawled initials. A signature means, "I helped and I agree." This physical declaration of ownership and participation helps formalize the importance of being involved and being responsible for what the group discusses, decides, creates, agrees upon, and learns. Such visible ownership of the group product also decreases the feeling of anonymity that can accompany group work and begins the accountability process. (In addition, the signature is a tool to use if a student says later, "Well, I didn't agree with what my group did." In such a case, the teacher responds, "Isn't this your signature? When you sign, you become responsible.")

In some instances, of course, groups cannot reach consensus. Students may then designate and sign the parts to which they do agree, and do the same for the parts they cannot negotiate.

Another fundamental way to ensure individual accountability is by holding every student responsible for being able to describe any aspect of the group activity. As previously indicated, the teacher should insist on group questions: every student must know in advance that the teacher, in responding to a group request for help, may have *any* student in the group be the one to ask the group question.

Similarly, the teacher may decide not to select students for the role of reporter for a given group activity until after the activity has been completed. In this way, all students must be ready to report on their group's findings by describing the group product, by explaining why it looks the way it does, by giving reasons for the group response, and so on.

When designating who will be the reporters, the teacher may choose one of three different selection methods. One method is random selection within each group, with one person selected at random to report from each group. Another method is random selection of a few names from an envelope, or a container that has slips of paper, one with the name of each student in the class. The teacher selects, say, three to seven reporters from the room, regardless of which group they represent. The teacher determines how many reporters to call on, based on the number of acceptable responses and what was observed during group work time. If responses are mostly correct, then the lesson can continue; if some responses are incorrect, the class may need to review selected items or explore further; if many responses are inappropriate, it may be necessary to revisit the investigation. A third method of selecting reporters involves the idea of shared roles: the teacher may randomly select a group or groups to report and have everyone in the group present part of the report.

No matter how the reporters are selected, to be fair and to motivate the students to pay attention to the task at hand, the method of selection, but not the identity of the reporters, should be announced before the activity begins.

Opportunities for individual accountability are an integral part of each lesson in *Contemporary Mathematics in Context*. At the "Checkpoint," for example, some students are called upon to report for their groups; random selection of reporters works well here. Following each "Checkpoint," another opportunity is

provided in the "On Your Own" task. After developing the mathematical ideas in groups and then summarizing or perhaps refining them in a class discussion, students individually complete the "On Your Own" task based on their understanding of the mathematics. This work is completed alone, without the assistance of the group, and it can be evaluated and recorded in a gradebook. This kind of accountability, too, should be decided upon and announced before work begins.

When students work in groups, it can be difficult for the teacher to know what an individual student participated in or learned. By including individual accountability in every activity, the teacher alerts the students to the fact that they will be held accountable, and the teacher thus increases the likelihood that students will pay attention to, and participate in, group work.

HANDLING CONFLICT EFFECTIVELY

A final area in which effective classroom management contributes to the collaborative learning process is in the handling of conflict that is counterproductive. The key to preventing such conflict lies in adherence to the rules for promoting respect. Even when rules for acceptable behavior have been discussed and posted, conflict can arise during group work. The amount of potential conflict varies indirectly with students' social skills. Students with a high degree of social skills will discuss, negotiate, and reach consensus with minimal conflict. When presented with new or different ideas, students with few social skills may use group work as a time to argue. The goal is to encourage healthy debate and to eliminate negative conflict.

To reduce further the likelihood of negative conflict, teachers may set the stage for group activities by initiating a discussion to check for understanding or by conducting a brainstorming session (without evaluation of the ideas contributed). The launch activity that begins each lesson serves this purpose.

Another way of preparing students to handle conflict is to have them finish and then discuss, either as a class or as groups, statements such as, "When we had a disagreement, it was helpful to. . . ." Also, role-playing can help students understand the differences between healthy debate and negative conflict and also help them practice handling conflict.

When a group seems to be at an impasse, the teacher should help the group resolve a conflict rather than simply breaking up the group. In resolving conflict, as in other aspects of cooperative learning, the teacher functions as guide or coach. Giving help does not mean entering into the conflict or solving the problem for the students. The teacher must help the students to become their own best problem solvers.

Conflict can be healthy and can stimulate thinking if teachers help students learn how to question each other with a positive tone; however, conflict resolution requires complex skills and will take time for students to practice and learn.

Conclusion

An atmosphere where students feel free to express their thoughts without derision from classmates, are encouraged to think deeply about mathematics, and are provided the opportunity to grow intellectually is truly a challenge to create. The culture created within the classroom is crucial to the success of this curriculum. It is important to inculcate in students a sense of inquiry and responsibility for their own learning. Without this commitment, active, collaborative learning by students cannot be effective. The percentage of ninth-grade students who begin school in the fall with these skills is very small. The work ethic and social skills of ninth graders is not ideal in many cases, but teachers indicate that with persistence this changes. In many cases, students returning to school as tenth graders seem to have miraculously matured over the summer and actually are eager to work collaboratively with classmates.

In order for students to work collaboratively, they must be able to understand the value of working together. Some students seem satisfied with the rationale that it is important in the business world. Others may need to understand that the struggle with verbalizing their thinking, listening to others' thinking, questioning themselves and other group members, and coming to an agreement increases their understanding and retention of the mathematics as well as contributes to forming important thinking skills or habits of mind.

This level of understanding the importance of collaborative groups is particularly important for students who are inclined to quickly and superficially cover the mathematics and move on. They often resist thinking deeply about the mathematics and fail to see the value in a multiple-perspective approach. Once these students understand that collaborative work broadens and deepens their own mathematical understanding, they can exert a positive influence on their classmates.

Issues of helping students to work collaboratively will become less pressing as both you and your students experience this type of learning. You may find it helpful to refer back to this reference guide and discuss effective cooperative groups with colleagues a few weeks into the semester.

Further Reading

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ASSESSMENT

Throughout the *Contemporary Mathematics in Context* curriculum, the term "assessment" is meant to include all instances of gathering information about students' levels of understanding and their disposition toward mathematics for purposes of making decisions about instruction. The dimensions of student performance that are assessed in this curriculum (see chart below) are consistent with the assessment recommendations of the National Council of Teachers of Mathematics *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1988) and the *Assessment Standards for School Mathematics* (NCTM, 1995). They are much broader than those of a typical testing program.

ASSESSMENT DIMENSIONS

Process	Content	Attitude
Problem Solving	Concepts	Beliefs
Reasoning	Applications	Perseverance
Communication	Representational Strategies	Confidence
Connections	Procedures	Enthusiasm

Sources of Assessment Information

Several kinds of assessment are available to teachers using *Contemporary Mathematics in Context.* Some of these sources reside within the curriculum itself; some are student-generated; and some are supplementary materials designed specifically for assessment. Understanding the nature of these sources is a prerequisite for establishing guidelines on how to score assessments, making judgments about what students know and are able to do, and assigning grades.

CURRICULUM SOURCES

Two features of the curriculum, questioning and observation by the teacher, provide fundamental and particularly useful ways of gathering assessment information.

Questions and Questioning The student texts use questions to facilitate student understanding regarding new concepts, how these concepts fit with earlier ideas and with one another, and how they can be applied in problem situations. Whether students are working individually or in groups, the teacher is given a window to watch how the students think about and apply mathematics as they attempt to answer the questions posed by the curriculum materials. In fact, by observing how students respond to the curriculum-embedded questions, the teacher can assess student performance across all process, content, and attitude dimensions described in the chart above.

Student responses to questions such as "What if . . . ?", "Why?", and "Can you explain your response?" provide insights into how students reason about and communicate mathematics. How well students see connections among mathematical ideas and their applications can be assessed with questions such as "How is this like [an earlier idea] and how is it different?" A question such as "What do you predict will happen?" may provide insights not only into students'

understanding of the relevant content, but also into their dispositions toward mathematics. These are just some of the types of questions that can provide good assessment information to help teachers make appropriate instructional decisions. Although such questions are commonplace in the instructional materials, similar questions can be framed by teachers in order to probe student understanding further. While observing individual students as they respond to such questions, teachers often take notes or complete checklists to help judge the growth of individual students or to provide detailed information for grade reports.

Specific features in the student material that focus on different ways students respond to questions are the "Checkpoint," "On Your Own," and MORE *(Modeling, Organizing, Reflecting,* and *Extending)* sets at the end of each investigation. "Checkpoint" features are intended to bring students together, usually after they have been working in small groups, so they may share and discuss the progress each group has made during a sequence of related activities. Each "Checkpoint" is intended to be a whole-class discussion, so it should provide an opportunity for teachers to assess informally the levels of understanding that the various groups of students have reached.

Following each "Checkpoint," the "On Your Own" tasks are meant to be completed by students working individually. Student responses to these tasks provide an opportunity for teachers to assess the level of understanding of each student.

Activities in the MORE sets serve many purposes, including post-investigation assessment. As discussed on page 13 in this guide, each type of activity in a MORE set has a different instructional purpose. Modeling tasks help students demonstrate how well they understand and can apply the concepts and procedures developed in an investigation. Organizing activities demonstrate how well students understand connections between the content of an investigation and other mathematical and real-world ideas. In-class discussions based on Organizing tasks are a crucial step in assisting students' development of a full understanding of the mathematical content and connections. Reflecting activities provide insights into students' beliefs, attitudes, and judgments of their own competence. Extending tasks show how well students are able to extend the present content beyond the level addressed in an investigation. The performance of students or groups of students in each of these types of activities provides the teacher with further information to help assess applicability, connectedness, and depth of the students' evolving understanding of mathematics.

Finally, an opportunity for group self-assessment is provided in the last element of each unit, the "Looking Back" lesson. These activities help students pull together and demonstrate what they have learned in the unit and at the same time provide helpful review and confidence-building for students.

Observation The other fundamental method of assessment that is built into use of *Contemporary Mathematics in Context* is observation by the teacher. Good teachers, of course, have always learned a great deal by observing individual students as they do mathematics. Such observation continues to be a valuable source of assessment information, and because many activities are

completed by students working in small groups, teachers using these materials have many opportunities to observe the performance of students as they work with others.

Typically, observations made by the teacher focus on student progress in understanding and applying the mathematical content of the lessons. However, in today's society, working well with others in problem-solving groups is extremely important, and developing students' ability to do so is an important goal of *Contemporary Mathematics in Context.* Thus teachers also should observe and assess behaviors of group members such as those listed here.

- Communicating mathematical ideas to other members of the group
- Dividing a task fairly among the group members
- Agreeing on a structure for completing a task
- Taking time to ensure that all group members understand
- Recording results regularly and clearly
- Using, in appropriate ways, the suggestions and ideas of all group members
- Fairly representing a group consensus
- Pushing the group to think deeply about the mathematics under investigation
- Reporting the group's progress to the whole class in a complete and interesting way

STUDENT-GENERATED SOURCES

Other possible sources of assessment information are writings and materials produced by the students in the form of student journals and portfolios.

Student Journals Student journals are notebooks in which students are encouraged to write briefly, but frequently, their personal reflections concerning the class, the mathematics they are learning, and their progress. These journals are an excellent way for the teacher to gain insights into how individual students are feeling about the class, what they do and do not understand, and what some of their particular learning difficulties are. This information can be very useful for planning instruction that will meet the needs of individual students in the class. For many students, the journal is a nonthreatening way to communicate with the teacher about matters that may be too difficult or too time-consuming to talk about directly. Journals also encourage students to assess their own understanding of, and disposition toward, the mathematics they are studying.

One effective approach to the use of journals is for the teacher to provide prompts (*i.e.*, questions or statements) to which the students give a written response in their journals. The Reflecting activities in the MORE sets include many good prompts for journal entries. Such prompts may be given two or three times a week, and the writing may be done during the last few minutes of class or at home. Occasional journal prompts occur following the "Checkpoints."

The teacher should collect, read, and respond to each journal at least once a month—more often, if possible. Teacher responses to the journal entries should

be nonjudgmental statements of interest or questions of clarification. Many teachers stagger their journal collection, reviewing perhaps one-fourth of them each week. Teachers are encouraged to have their students keep journals and write in them regularly.

Portfolios A portfolio is a collection of a student's work accumulated over time. Typically, portfolios provide a tool for assessing one or more of the following outcomes: student thinking, growth over time, mathematical connections, student's views of herself or himself as a mathematician, and the problem-solving process as employed by the student. An excellent, practical source about the use of portfolios in mathematics is the NCTM publication, *Mathematics Assessment: Myths, Models, Good Questions, and Practical Suggestions.*

The *Contemporary Mathematics in Context* assessment program provides many items that would be appropriate for a student's portfolio, including reports of individual and group projects, journal excerpts, teacher-completed observation checklists, end-of-unit assessments (especially the take-home activities), and the extended cumulative projects. One way students can develop a portfolio is to collect all their written work in a folder, sometimes called a working portfolio. Then at least once each semester, students go through their working portfolio and choose items that they think best represent their growth during this time period. After writing a paragraph or two explaining why each piece of work was chosen, each student puts the chosen items with the written rationales into a new folder that becomes the actual portfolio.

SUPPLEMENTARY ASSESSMENT MATERIALS

The *Contemporary Mathematics in Context* instructional materials include for each unit a third source of assessment information—a supplement that contains end-of-lesson quizzes and end-of-unit assessments in the form of an in-class unit exam and a take-home assessment. Calculators often are required and it is intended that they always are available to students. Since many rich opportunities for assessing students are embedded in the curriculum itself, you may choose not to use all the lesson quizzes for each unit.

End-of-Lesson Quizzes Two forms of a quiz covering the main ideas of each lesson are provided. These quizzes, which are the most traditional of all the assessment methods and instruments included with the *Contemporary Mathematics in Context* materials, are comprised of fairly straightforward problems meant to determine if students have developed understanding of the important concepts and procedures of each lesson. Thus, the quizzes focus on the content dimension of the Assessment Dimensions.

End-of-Unit Assessments At the end of each unit, the teacher may choose to use an in-class exam or a take-home assessment. Both measure process and content outcomes, and the take-home assessments also provide good information concerning students' dispositions toward mathematics. Teachers are encouraged to try each kind of assessment sometime during the year.

The in-class exams, two versions of which are provided, are intended to be completed in a 50-minute class period by students working individually. Teachers should decide whether to allow students to have access to the textbook and/or their Math Toolkit during in-class exams. Occasionally allowing students to have class notes available for the exams also may motivate them to keep their notes well organized. In general, if the goals to be assessed are problem solving and reasoning, while memory of facts and procedural skill are of less interest, resources should be allowed. However, if automaticity of procedures or unaided recall are being assessed, it is appropriate to prohibit resource materials.

The two forms of each in-class exam are *not* equivalent, although they assess essentially the same mathematical ideas. Teachers should preview the two versions carefully before making a choice, and they should feel free to revise or delete items and add new ones if necessary. Students who do poorly on one form of the exam should be given feedback, and, after additional review and preparation, may be offered the second form of the exam, at the teacher's discretion.

For the take-home assessments, five possible assessment activities are included. The students or the teacher should choose one or, at most, two of these activities. These assessments, some of which are best done by students working in pairs or small groups, provide students with the opportunity to organize the information from the completed unit, to extend the ideas of the unit into other areas of interest to them, to work with another student or group of students, and to avoid the time pressure often generated by in-class exams. For students whose learning styles and dispositions put them at a disadvantage on in-class exams, this kind of assessment allows them to demonstrate their grasp of some of the major ideas in the unit in a culminating, holistic activity.

Of the five activities in a take-home assessment package, the first three require a day or two for one or two students to complete. The intent is for students to revisit and extend the material covered in the unit by designing their own test for the unit or by producing a short summary report in which graphs, tables, charts, data, and other mathematical material are used. The test or report is then turned in or presented to the class.

The last two activities of a take-home assessment package are extended projects that require up to a week for students, usually working in groups, to complete. The group is required to conduct investigations that apply main ideas from the unit in ways that go beyond what students have done in class. The group then produces a report (or presentation) that synthesizes and evaluates the complex body of information gathered. Each such activity is of sufficient depth that the group is required to develop a plan to separate tasks so that the entire activity can be completed successfully in the time allowed. It is recommended that, on one or two occasions during the time that students are working on these assessments, the teacher use some class time to check on their progress and to give guidance and encouragement.

Extended Cumulative Projects Assessment traditionally has been based on evaluating work that students have completed in a very short time period and under restricted conditions. Some assessment, however, should involve work done over a longer time period—a week or more—and with the aid of resources. Thus an extended assessment project is included with the Unit 3 assessments for

each course. These projects are investigations that make use of some of the main ideas encountered in the curriculum up to that time. The activities, which are intended to be completed by small groups of students in a week or more, are similar to the extended take-home assessment projects at the end of each unit (see above), except that they require understanding of mathematical content from several units. It is recommended that, during these extended projects, the teacher check student progress every few days and give students opportunities to use teacher feedback on at least one draft as they prepare their final report.

Scoring of Assessments

High expectations of the quality of students' written work will encourage students to reach their potential. You will notice the solutions to investigations and MORE sets sometimes indicate that "responses may vary." This is used when more than one correct response is acceptable. Students always should have reasonable explanations for their responses. The quality of the responses will vary and some responses may simply be incorrect. Model written responses to "compare," "describe," and "explain" tasks are developed in Lesson 1 of Unit 1 in Course 1. Some teachers use quality student responses as model responses for the class throughout the course. Continued emphasis on reasonable explanations helps students develop their thinking and communication skills throughout their mathematics studies.

Assigning scores to open-ended assessments and to observations of students' performance requires more subjective judgment by the teacher than does grading short answer or multiple-choice tests. It is therefore not possible to provide a complete set of explicit guidelines for scoring open-ended assessment items and written or oral reports. However, the following general guidelines may be helpful. (Adapted from *Mathematics Assessment: Myths, Models, Good Questions, and Practical Suggestions*, copyright 1991 by the National Council of Teachers of Mathematics. All rights reserved.) Teachers can use the general framework to develop guidelines for specific assessment items or activities.

OPEN-ENDED ITEMS

When scoring the student work on open-ended assessment tasks (as opposed to short answer or multiple-choice, for example), the goal is to reward in a fair and consistent way the kinds of thinking and understanding that the task is meant to measure. Preparing to score open-ended assessment tasks is best done using a three-step process. First, teachers should have a *general rubric*, or scoring scheme, with several response levels in mind. As the name suggests, the general rubric is the foundation for scoring across a wide range of types of open-ended tasks. The following general rubric can be used for most assessment tasks provided with the *Contemporary Mathematics in Context* materials.

To score a particular task in a reasonably unambiguous, fair way, the teacher will need a *specific rubric* refining the general score level to the task at hand. This rubric should take into consideration the ways in which students construct their responses to that task. The specific rubric is the second step in the three step process.

Finally, since verbal definitions for score levels can never be entirely unam-

biguous, the teacher will need to identify examples of student work to anchor the specific rubric. These examples are often referred to as *anchor items*. The anchor items for each scoring level should be chosen to illustrate the range of responses that are typical of students who score at that level. In the following, the complete process for scoring is described using a sample open-ended task and examples of actual student work for *Contemporary Mathematics in Context*.

GENERAL SCORING RUBRIC

For most open-ended mathematics tasks, credit can be assigned in decreasing amounts according to the following levels of performance. This general rubric has five scoring levels corresponding respectively to 0 to 4 points, but three or four levels may work better for some tasks. Adjustments to the rubric for the number of levels can be made in obvious ways. Not all parts of the description of each level will be relevant to every open-ended task.

4 points Contains complete response with clear, coherent, and unambiguous explanation; includes clear and simple diagram, if appropriate; communicates effectively to identified audience; shows understanding of question's mathematical ideas and processes; identifies all important elements of question; includes examples and counterexamples; gives strong supporting arguments

3 points Contains good solid response with some, but not all, of the characteristics above; explains less completely; may include minor error of execution but not of understanding

2 points Contains complete response, but explanation is muddled; presents incomplete arguments; includes diagrams that are inappropriate or unclear, or fails to provide a diagram when it would be appropriate; indicates some understanding of mathematical ideas, but in an unclear way; shows clear evidence of understanding some important ideas while also making one or more fundamental, specific errors

1 point Omits parts of question and response; has major errors; uses inappropriate strategies

0 points No response; frivolous or irrelevant response

A specific rubric is the result of rewriting the definitions of the scoring levels in the general rubric so they explicitly account for the details of the assessment task at hand and to the range of actual student responses to that task. As an illustration, consider the following task.

Sample Assessment Task The length s, in inches, of a spring is a function of the weight w, in pounds, that is attached to the spring. For a particular spring, the modeling equation is:

$$s = 20 + 5w$$

a. Explain what the 20 in the equation tells you about the spring.
b. Solve the following equation and show your work: $36 = 20 + 5w$.
c. Explain what the solution in Part b tells you about the spring.
d. How long is the spring if you place a weight of 1.2 pounds on it?

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The process of developing a specific rubric for this task involves an initial decision concerning whether to score the entire task as a unit or to score each part separately. In this case, it could be reasoned that the parts were sufficiently different to make scoring each part the better decision. Following that decision, consider Part a by first asking what a top level (4 point) response should contain. After specifying the characteristics of a solution required for 4 points, next ask what a minimal solution must contain; that is, what must the response include for a score of 1 rather than 0. Identifying the "best" and "least" acceptable solution can usually be done without reference to student work, although some minor changes in these initial definitions may need to be made later to account for some unanticipated student responses.

Distinctions between scores of 4 and 3, 3 and 2, and 2 and 1 are more difficult. It is helpful to make an initial attempt at defining scores of 3 and 2, using only an analysis of the item and what are likely responses, but then to move quickly to examining a set of student work. Actual student responses help refine definitions, forcing decisions about what should count in each scoring category. Refinements can be made to account for unexpected student responses while keeping in mind the general rubric and the goal to reward in a fair and consistent way the kinds of thinking and understanding that the task is meant to measure.

By repeating the process described in the previous paragraph for Parts b through d, a specific rubric similar to the following would be constructed. Recall that a score of 0 is given for either no response or for a completely irrelevant response.

SPECIFIC SCORING RUBRIC

Part	4 points	3 points	2 points	1 point
a	Initial length of spring *or* lengh when no weight is attached	Correct answer is implied, but answer is vague, *e.g.*, "What the spring started at."	Confusion between weight and stretch *or* incorrect response, but containing word synonymous with weight or stretch, *e.g.*, "How long the spring is."	Incorrect response that shows some relevance to a spring, length, weight, stretch, or 20
b	Correct answer (3.2 or 16/5) with correct work, either algebraic or calculator	Correct answer, but no solution or wrong steps in the solution process *or* correct solution, but arithmetic error leads to incorrect answer	Incorrect numerical answer with flawed, but potentially or partially correct, solution process	Incorrect numerical answer with no work or inappropriate work *or* incomplete work that seems to be partially correct, but no answer is given
c	Correct answer similar to, "When weight of 3.2 lbs is attached to the spring, its length is 36 in."	Right idea, but missing labels make distinction between weight and length unclear *or* relationship between length and weight is mentioned, but answer is vague	Mentions at least one of 3.2 and 36 and at least one of lbs and in (or length and weight), but does not connect them or switches them	An attempt that mentions 36 or 3.2, but not both *or* an attempt that is in some way relevant to a spring, weight or length
d	Writes the equation $s = 20 + 5(12)$ correctly and gets the answer 26, that is, both work and answer are correct	Correct answer, 26, but no work or an equation that is incorrect *or* a correct equation with an arithmetic error leading to an incorrect answer	Incorrect answer with correct equation, but work has one or more algebraic errors (not just arithmetic)	Incorrect numerical answer with wrong equation, inappropriate work, or no work

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ANCHOR ITEMS

The following responses demonstrate the range of responses to Part a at each score level.

Score	Student Responses to Part a
4	(1) "That the spring is 20 inches long before adding weight to it." (2) "The spring is 20 inches long to start with."
3	(1) "What the spring started at." (2) "20 tells me that the spring has to be at least 20 inches long."
2	(1) "It tells me the length in inches." (2) "It explains that there are 20 inches in the spring."
1	(1) "That you can have anything you want be equal to 20." (2) "The spring is 20 more than twice the weight." (3) "20 inches"

Note that a score of 4 requires explicit mention of 20 inches being the length of the spring at the start or with no weight attached. The responses that were scored 3 suggest that the students may have had the correct idea, but they failed to express it explicitly. Most responses in the 2-point category simply said the spring is 20 inches long, an incomplete response, but at least it identifies 20 with the length of the spring. The responses assigned a score of 1 failed to relate 20 to the length of the spring and, in some cases, gave a clearly incorrect description of the role of 20.

The responses in the chart below are meant to clarify the rubric for Part b. (The examples are numbered for identification in the discussion that follows.)

SCORES FOR STUDENT RESPONSES

Scores for Part b

4	(1) $36 = 20 + 5w$ $-20 \quad -20$ $\frac{16}{5} = \frac{5w}{5}$ $w = 3.2$	(2) $36 - 20 = 16$ $5\overline{)16}$ $\quad 3.2$ $w = 3.2$	(3) Go to Y = menu and enter $20 + 5x$. Use the table. Press the down arrow until you reach 36 in the Y_1 column. Look across from 36 to get $w = 3.2$
3	(1) $36 - 20 = 20 - 20 + 5w$ $\frac{16}{5} = \frac{5w}{5}$ $w = 3.1$	(2) $w = 3.2$	
2	(1) $20 + 36 = 20 - 20 + 5w$ $56 = 5w$ $\frac{56}{5}$ $w = 11.2$	(2) $36 - 20 = 20 - 20 + 5w$ $16 \div 5 = 5 - 5w$ $w = 11$	
1	(1) $20 + 36 = 20 - 20 + 5w$ $56 = 20w$ $\frac{56}{5}$	(2) $\frac{5}{5w} = 5$ $25 + 5 + 36$ $w = 66$	

The three main categories of responses that are assigned a score of 4 are illustrated by the examples in the table on page 43. In sample response (1), the standard paper-and-pencil approach to solving a linear equation is clearly applied correctly. In sample response (2), the student showed the computations, but not what was done. While response (2) is a less complete solution than response (1), it was judged as including the correct answer and the correct work and so assigned a score of 4. A typical correct calculator solution is described in example response (3). Since *Contemporary Mathematics in Context* emphasizes the use of a graphics calculator, this solution also receives full credit.

In the examples receiving a score of 3, an arithmetic error in an otherwise correct solution is shown in sample response (1), and the correct answer only with no work is response (2). For a score of 2, the student in the first example erroneously added 20 to the left side, but then correctly subtracted 20 from the right side and finally correctly divided by 5. Similarly, in the second example that was scored 2, the student correctly subtracted 20 from both sides, but then got off-track by subtracting 5 (thinking that $5 - 5w = w$). The first example that was scored 1 is very similar to the first example that was scored 2. The work is partially correct in both cases, but the student who received the score of 1 did not indicate a value for w. The second example that was scored 1 has a numerical answer, but no part of the work seems to be on target.

No sample work is provided for Parts c and d. You and a colleague may wish to examine the rubric for Parts c and d carefully in light of the foregoing examples of responses to Parts a and b and generate examples of student work for each part that would receive any score from 1 to 4.

REPORTS AND PRESENTATIONS

Four or five levels of holistic scoring may be used for reports and presentations. At each level the teacher must be able to identify the key prerequisite characteristics of a report, such as a particular table, graph, diagram, or function. The teacher also will need to judge the quality of the student's reasoning, problem solving, and communication skills as displayed in the report or presentation. Holistic scoring by the teacher or by classmates could also be used to assess individual or group presentation of work on the Organizing tasks.

An alternative to the holistic approach involves making separate judgments of the student's reasoning, problem solving, communication skills, and perhaps other categories that may be appropriate for a given report. Then, weighted in a manner that the teacher deems appropriate, these scores may be combined to determine a single grade for the report. Whether scoring in different categories is better than holistic scoring is debatable, but breaking down the grade into components probably makes the overall grade more understandable to the student and easier for the teacher to defend.

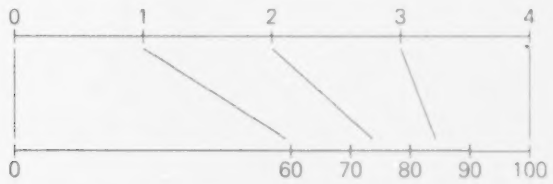
A more detailed look at evaluating extended projects is given in the teacher notes that accompany the extended project assignments.

Assigning Grades

Because the *Contemporary Mathematics in Context* approach and materials provide a wide variety of assessment information, the teacher will be in a good position to assign a fair grade for student work. With such a wide choice for assessment, a word of caution is appropriate: *it is easy to over-assess students, and care must be taken to avoid doing so.* A quiz need not be given after every lesson nor an in-class exam after every unit. The authors believe it is best to vary assessment methods from lesson to lesson, and from unit to unit. If information on what students understand and are able to do is available from their homework and in-class work, it may not be necessary to take the time for a formal quiz after each lesson. Similarly information from project work may replace an in-class exam.

Deciding exactly how to weigh the various kinds of assessment information is a decision that the teacher will need to make and communicate clearly to the students. Commonly, teachers assign a percentage of the final grade to different types of assessment such as in-class participation (dispositions), quizzes, exams, MORE tasks completed outside of class, and extended assessment activities. Sometimes teachers make more holistic judgments.

Some flexibility in grading is required. Recalling that a score of 1 on a 0–4 scoring rubric indicates some understanding and 2 indicates a considerable amount of progress on challenging problem solving and reasoning assessment tasks. Even though 1/4 is 25% and 2/4 is 50%, which typically might be equated with failing grades, the level of understanding actually may be worthy of a grade of D or C. In short, the usual 90% for A, 80% for B, and so forth may not work as a grading scale. Some teachers use a rubric to score quizzes and tests and then assign letter or percent grades on a curve. Others use another, slightly different approach in which they directly translate a rubric score to the usual scale. Since 0%–59% is the interval of failure on this scale, there are really five intervals: 0–59, 60–69, 70–79, 80–89, and 90–100. In this approach, the teacher maps the five intervals of a rubric score directly into these intervals, using the following scheme or some variation of it.



This direct translation approach may have a psychological advantage over grading on a curve, which often is perceived as little more than raising low grades. Instead, with direct translation, the teacher, students, and their parents simply (and quite legitimately) think of the rubric scores as equivalent to the corresponding scores on the percent scale.

Whatever the method for determining grades, the authors recommend the teacher take advantage of the multiple sources of assessment information available, and do not base grades merely on quizzes and end-of-unit exams. The following is typical of the way the pilot- and field-test teachers weighed factors

when assigning grades. Of course, individual teachers may decide on their own breakdowns.

Unit Tests	25%
Quizzes	25%
Homework	20%
Group work/participation	15%
Written/oral reports	10%
Notebooks/journals	5%

One grading issue in using *Contemporary Mathematics in Context* relates to group work. Every student working in a group has a responsibility to help the group operate smoothly and to facilitate the learning of other group members. These outcomes are very important in this curriculum, and assessing and encouraging them for each student is no less important. For grading group assignments which result in a single report, all group members receive the same grade. So how can individual grades be used to promote successful group work? One approach that seems to work well is to give extra credit to all group members when all members of a group do very well on a quiz or exam. Generally, this extra credit should be difficult, but not impossible, to earn.

Another grading issue arises from the fact that this curriculum is designed for *all* students. It intends to challenge the best students while being accessible to all. Fair grading of such a wide range of students is not easy. Schools may choose to have two grading scales, "regular" or "core" and "honors" or "Core-Plus." The latter category typically requires consistently high-quality work on the core topics plus work of similar quality on additional Extending tasks in every unit. A grade in the Core-Plus category could be assigned more weight than a corresponding grade in the core category in the computation of a student's grade point average. For example, as many schools do with Advanced Placement classes, an A in the Core-Plus category could count as 5 quality points whereas an A in the core category might count as 4.

TIPS ON STUDENT WORK AND WRITING

Most teachers require students to maintain a three-ring binder which contains their written work from investigations, "On Your Own," and MORE assignments. Written assessments and projects which have been returned to students may be organized in the three-ring binder or held in a student portfolio which remains in the classroom at all times.

In order to avoid being overwhelmed with paperwork and to avoid over-assessing students, teachers use a variety of sampling techniques. By using a selection of the options below, individual teachers can develop their own scheme for paperwork management.

1. Check one paper per group for each investigation.
2. Check only for completion of assigned work.
3. Check for understanding by using a *homework quiz.* (Students copy their already-written responses to selected homework tasks such as Modeling 1b or Organizing 4f, or they simply circle their responses and hand in these items.)
4. Check different tasks for different students. (Some teachers require students who wish to earn an A to do one or two Extending tasks with every MORE assignment. By assessing different items, teachers have a broader view of what students know and are able to do.)
5. Collect notebooks and grade for completion at the end of each unit.
6. Check for completion of homework by a quick survey while students work on an investigation.
7. Keep a file card for each student which lists tasks yet to be completed by each student. Students need to have all investigations and homework completed before taking a written assessment.
8. Collect papers randomly from about one-third of the class for each assignment. Grade and comment on responses to all questions, or just certain questions.
9. Have the students exchange papers in class as they go over the Organizing tasks from the MORE assignments. Students can address strengths and weaknesses of the responses and, with teacher guidance, assign a check, check-plus, or check-minus.
10. Collect papers from the entire class, check the papers for completeness, and then examine one or two questions or parts of questions for correctness, complete sentence responses, and thoroughness.

Some teachers also have students include a section (often containing colored paper) in the three-ring binder for the student constructed Math Toolkit. (See page 5.) Other teachers require a separate spiral notebook or a booklet made from grid paper folded in half with a colored cover. One advantage of a separate booklet is that students have long-term access to their reflections and compilations of class-generated definitions and mathematical concepts which they have developed throughout the year. If the Math Toolkit is kept in the three-ring binder, it should be maintained throughout the year even though other sections

of the notebook may be emptied when starting each new unit. Students should also continue constructing their Math Toolkits during all three courses of *Contemporary Mathematics in Context*. The toolkits give students a valuable reference as well as a long-term record of their mathematical growth.

It is important to help students develop good work habits. Previous mathematics programs may not have emphasized writing. Students studying *Contemporary Mathematics in Context* for the first time need to work on writing complete answers using complete sentences. Some students think that for a mathematics question, one number or one word is a sufficient answer. Throughout the course, students need to be encouraged to express their ideas completely and clearly. It is very important to have high expectations and hold students to high standards.

MAINTAINING SKILLS

The developers have identified a set of paper-and-pencil technical competencies that all students should acquire. To provide additional practice with these core competencies, a special maintenance feature is included in blackline master form in the *Teacher's Resources*.

Beginning with Unit 4 of Course 1, *Graph Models*, and then continuing with each unit thereafter, a supplementary set of maintenance tasks provides periodic review and additional practice of basic skills. These skills will be continually revisited to ensure mastery by each student at some point in the curriculum.

Use of the maintenance with Lesson 2 of each unit will allow students time to work on skills without interrupting the flow of the unit. Students should be well into the unit before it is distributed. You may wish to allow a few minutes at the end of a class period to revisit these skills with various groups of students who need assistance while other groups choose an extending task.

The maintenance materials prepared for each unit span technical competencies across each of the strands. In each case, the first presented task is a contextual problem, but the remaining tasks are not contextualized. Students should *not* use a calculator for theses tasks unless so directed.

Communicating with Parents

Parents are understandably concerned about the implementation of any major curriculum or instructional changes in programs that affect their children. Their understanding and support will play an important role in the successful implementation of the *Contemporary Mathematics in Context* curriculum. This understanding and support can also be a big factor in their children's success with mathematics. For many parents, the traditional sequence of algebra 1, geometry, algebra 2, and precalculus courses was the gateway to college. The Core-Plus Mathematics Project curriculum provides an alternative, contemporary, *Standards*-based route to both college and careers in the technology-based workplace of the 21st century.

It is important that parents are provided information about *Contemporary Mathematics in Context* well in advance of its implementation and periodically while it is in use. Schools participating in the testing and evaluation of the curriculum organized family "Math Nights" in which parents were provided a rationale for the proposed changes in curriculum and instructional practices, including expectations of colleges, small businesses, and industry; were guided through the mathematics of the curriculum; and were given opportunities to express concerns. Opportunities for parents to review the textbooks permit them to see that the texts include important ideas and methods of algebra, geometry, and functions—and much more: statistics, probability, trigonometry, and discrete mathematics. Parents also see in the texts the kinds of mathematics that they themselves use in their jobs.

As noted in the next section on college admissions, students participating in the preliminary testing of the *Contemporary Mathematics in Context* curriculum have been admitted to a wide variety of colleges and universities. This information is helpful in relieving concerns of some parents. It is also important to note that increasing numbers of colleges and universities are offering "reform" calculus programs that share many of the features of this curriculum: emphasis on developing conceptual understanding and presenting problems in context, use of technology tools such as graphing calculators or computers, collaborative group work, student projects, and writing in and about mathematics.

As an initial step toward communication, you might consider sending periodic information letters to parents. Two sample letters for Course 1 follow.

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[school letterhead]

[date]

Dear Parent:

We would like to share with you information about our new mathematics program. The program was developed by the Core-Plus Mathematics Project through a grant from the National Science Foundation. The texts, *Contemporary Mathematics in Context*, are based on national standards for curriculum and teaching developed by the National Council of Teachers of Mathematics and endorsed by 15 mathematical sciences organizations. The texts also reflect the needs of business and industry today who are calling for workers who can think and reason about quantitative situations, who can communicate effectively, and who can work together in teams.

The first two units of Course 1 are devoted to the study of important and broadly useful topics in statistics and algebra.

> In Unit 1, *Patterns in Data*, students learn to organize data using various graphical displays (histogram, number line plot, stem-and-leaf plot, box plot, scatterplot) and to summarize data using measures of center (mean, median, mode) and measures of variability (range, interquartile range, mean absolute deviation, percentiles). The last lesson, Relationships and Trends, introduces the idea of describing paired data using algebraic equations and inequalities. For the kinds of problems your son or daughter should be able to solve after completing the unit, see tasks 1 and 2 on pages 94 and 95 of the textbook.
>
> In Unit 2, *Patterns of Change*, students extend their understanding and skill in algebra in three ways. They learn how to recognize relationships among independent and dependent variables in problems and experiments and describe patterns in quantitative variables that change over time. They learn how to read and construct data tables and graphs that display relationships among variables. They begin developing symbol sense—the ability to connect important patterns of change to linear, exponential, and quadratic equations. After completing this unit, your son or daughter should be able to solve problems like 1 and 2 on pages 154 and 155 of the text.

Your son or daughter should be developing a Math Toolkit which summarizes concepts, facts, skills, and methods which he or she is learning. You may wish to review his or her math toolkit from time to time or consult it as you are providing assistance on homework.

We believe that through the CPMP materials your son or daughter will develop a new excitement for mathematics and will grow confident in his or her ability to think mathematically. You can contribute to this growth by supporting completion of work that is assigned to be completed outside of class.

If you have any questions about our mathematics program, do not hesitate to contact us.

Sincerely yours,

[Local principal, mathematics department chair, teacher]

[School letterhead]

[date]

Dear Parent:

This letter is a follow-up to our previous correspondence with you regarding our new mathematics program. The course in which your son or daughter is enrolled is the first of four courses designed to prepare students for success in college, in careers, and in daily life in contemporary society.

In the remaining units in the first course, students will continue to grow in their understanding and use of algebra, develop concepts and skills in geometry and in related areas of graph theory, and learn methods similar to those used in science, business, and industry for answering questions involving probability. Ideas from statistics are interwoven in many of these units.

In Unit 3, *Linear Models*, students learn how to recognize situations in which key variables change at a constant rate. They learn how to express and interpret those patterns of change in data tables, slopes and intercepts of straight-line graphs, and equations in the form $y = a + bx$. They learn techniques for solving linear equations and inequalities that arise in science and business problems. For the kinds of problems your son or daughter should be able to solve after completing this unit, see tasks 2, 3, and 5 on pages 244-247 of the textbook.

In Unit 4, *Graph Models*, students learn basic concepts of graph theory and they solve problems related to many different types of networks, including communication, computer, transportation, and distribution networks. Upon completion of this unit, your son or daughter should be able to solve problems like 1 and 2 on pages 321 and 322.

In Unit 5, *Patterns in Space and Visualization*, students continue their development of geometric concepts and skills. Students define, identify, and sketch prisms, pyramids, cones, and cylinders. They learn to apply the concepts and computational formulas for perimeter, area, and volume of plane and three-dimensional shapes. For the kind of problem your son or daughter should be able to solve after completing this unit, see task 1 on pages 416 and 417 of Part B of *Contemporary Mathematics in Context*.

In Unit 6, *Exponential Models*, students learn how to construct and use data tables, graphs, and equations in the form $y = a(b^x)$ to describe and solve problems about exponential relationships such as population growth, investment of money, and decay of medicines and radioactive materials. Upon completing this unit, your son or daughter should be able to solve problems like 2 and 3 on pages 478 and 479.

In Unit 7, *Simulation Models*, students learn to design and carry out simulations to estimate answers to questions about probability. See task 1 on pages 526 and 527 for a sample of the kind of problem your son or daughter should be able to solve after completing this unit.

As your son or daughter progresses through these units, continue to support their completion of homework. This outside-of-class work is essential to developing mathematical competence. And please do not hesitate to contact us if you have questions about the program.

Sincerely yours,

[Local principal, mathematics department chair, teacher]

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College Admissions

College Admissions

Since most college-preparatory curricula in the United States historically have been sequences of algebra 1, geometry, algebra 2, and precalculus, implementation of *Contemporary Mathematics in Context: A Unified Approach* may raise questions related to college or university admissions for students who have successfully completed three or four years of this program.

Although *Contemporary Mathematics in Context* is a relatively new high school mathematics program, integrated high school mathematics programs are not a new phenomenon. The New York State Regents Examinations have, for the last ten years, been based on an integrated mathematics curriculum. Graduates of that curriculum have been admitted to colleges and universities across the United States. Today, most higher education admissions offices are aware of the emerging integrated mathematics curricula being developed by projects, such as the Core-Plus Mathematics Project, funded by the National Science Foundation. The NCAA clearinghouse for student-athlete eligibility has also examined *Contemporary Mathematics in Context* and considers it suitable for college-bound student athletes. Admissions staffs and the NCAA typically review applications based on short course descriptions of the mathematical content rather than on course titles.

In cases where mathematics departments or admissions offices have reviewed descriptions of *Contemporary Mathematics in Context: A Unified Approach*, Courses 1–3, these courses have been approved as meeting the mathematics admissions requirements of the institutions. High school counselors can submit copies of the course descriptions as found on pages 6–11 or summary paragraphs based on these course descriptions. Such course descriptions when accompanying transcripts showing Integrated Mathematics or Unified Mathematics (1, 2, 3, 4) appear to be sufficient for admissions purposes. For student-athletes, the NCAA considers Course I to satisfy Level I requirements for student eligibility. Courses II and III satisfy Level II requirements.

Teachers of *Contemporary Mathematics in Context* report that their students have been accepted into a wide variety of higher education institutions, including the University of California, Berkeley, Northwestern University, Harvard University, the University of Chicago, and many others, some of which are listed on the following pages.

A Sampling of Colleges That Have Accepted CPMP Students

Alabama
- Auburn University
- Samford University
- University of Alabama
- University of Southern Alabama

Alaska
- University of Alaska, Fairbanks

Arizona
- Arizona State University
- Northern Arizona University
- University of Arizona

California
- California State Polytechnic University, Pomona
- California State Polytechnic University, San Luis Obispo
- California State University, Chico
- California State University, Long Beach
- California State University, Northridge
- California State University, San Bernardino
- Humboldt State University
- Palomar Community College
- Point Loma Nazarene College
- Saddleback College
- San Diego State University
- San Francisco State University
- San Jose State University
- San Marcos State University
- Sonoma State University
- Southwestern Community College
- University of California, Berkeley
- University of California, Davis
- University of California, Irvine
- University of California, Los Angeles
- University of California, Riverside
- University of California, San Diego
- University of California, Santa Barbara
- University of California, Santa Cruz
- University of San Diego
- University of Southern California
- Westmont College

Colorado
- Colorado Aero Tech
- Colorado School of mines
- Colorado State University
- Fort Lewis College
- Metropolitan State College of Denver
- U.S. Air Force Academy
- University of Colorado, Boulder
- University of Colorado, Colorado Springs
- University of Northern Colorado
- Western State College of Colorado

Connecticut
- University of Connecticut

Delaware
- University of Delaware

Florida
- Daytona Beach Community College
- Embry-Riddle Aeronautical University
- Florida A & M University
- Florida State University
- Lynn University
- Pensacola Junior College
- Rollins College
- Stetson University
- University of Florida
- University of Miami
- University of South Florida

Georgia
- Agnes Scott College
- Augusta State University
- Berry College
- Columbus State University
- DeKalb College
- DeVry Institute
- Emory University
- Gainesville College
- Georgia Baptist College of Nursing
- Georgia College and State University
- Georgia Institute of Technology
- Georgia Southern University
- Georgia Southwestern College
- Georgia State University
- Gwinnett Tech
- Life College
- Middle Georgia Junior College
- Morehouse College
- Morris Brown College
- North Georgia College
- Oglethorpe University
- Piedmont College
- Reinhardt College
- Savannah College of Art & Design
- Southern College of Technology
- Spelman College
- Truett McConnell College
- University of Georgia
- Valdosta State University
- West Georgia College
- Young Harris College

Hawaii
- University of Hawaii

Idaho
- Boise State University
- Ricks College
- University of Idaho

Illinois
- Barat College
- Columbia College
- DePaul University
- Greenville College
- Illinois Institute of Technology
- Lake Forest College
- Monmouth College
- North Central College
- Northwestern University
- Principia College
- University of Chicago

Indiana
- Ball State University
- Butler University
- Depauw University
- Goshen College
- Indiana Institute of Technology
- Indiana State University
- Indiana University, Bloomington
- Indiana Wesleyan University
- Purdue University
- Taylor University
- Tri State University
- University of Evansville
- University of Notre Dame
- Valparaiso University

Iowa
- Cornell College
- Graceland College
- Iowa Central Community College
- Iowa State University
- Kirkwood Community College
- Loras College
- Luther College
- Mercy/St. Lukes School of Radiologic Technology
- Mt. Mercy College
- Simpson College
- University of Dubuque
- University of Iowa
- University of Northern Iowa
- Wartburg College

Kansas
- Midamerica Nazarene College

Kentucky
- Georgetown College
- University of Louisville
- University of Kentucky

Maryland
- U.S. Naval Academy

Massachusetts
- Boston College
- Boston University
- Harvard University

Michigan
- Adrian College
- Albion College
- Alma College
- Aquinas College
- Baker College
- Calvin College
- Central Michigan University
- Concordia College
- Cornerstone College
- Davenport College
- Delta College
- Eastern Michigan University
- Ferris State University
- Glen Oaks Community College
- Great Lakes Christian College

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(*Michigan*, cont.)

GM Engineering and Management Institute
Grand Rapids Community College
Grand Valley State University
Hillsdale College
Hope College
ITT Tech
Jackson Community College
Kalamazoo College
Kalamazoo Valley Community College
Kellogg Community College
Kendall College of Art & Design
Lake Michigan College
Lake Superior State University
Lansing Community College
Lawrence Technological University
Macomb Community College
Michigan State University
Michigan Technological University
Monroe County Community College
Mott Community College
Muskegon Community College
Northern Michigan University
Northwestern Michigan College
Northwood University
Oakland University
Olivet College
Saginaw Valley State University
St. Mary's College
Siena Heights College
Spring Arbor College
University of Detroit, Mercy
University of Michigan
University of Michigan-Dearborn
University of Michigan-Flint
Washtenaw Community College
Wayne State University
Western Michigan University

Minnesota
Bemidji State University
University of Minnesota

Missouri
Evangel College
Lindenwood College
St. Louis University
Washington University

Montana
Montana State University
University of Montana

Nevada
University of Nevada, Las Vegas
University of Nevada, Reno

New Hampshire
Keene State College
University of New Hampshire

New York
Clarkson University
Cornell University
Culinary Institute of America
Fashion Institute of Technology
Ithaca College
School of Visual Arts
St. John's University

North Carolina
Appalachian State University
Brevard College
Catawba College
East Carolina University
Elon College
University of North Carolina, Charlotte
University of North Carolina, Greensboro
University of North Carolina, Wilmington
Wake Forest University
Western Carolina University
Winston-Salem State University

North Dakota
University of North Dakota

Ohio
Ashland University
Bowling Green State University
Capital University
Cedarville College
Columbus College of Art & Design
Danson University
Denison University
Heidelberg College
John Carroll University
Kent State University
Kenyon College
Kettering University
Malone College
Marietta College
Miami University
Mount Union College
Northwestern College
Oberlin College
Ohio State University
Ohio University
Owens Community College
University of Akron
University of Cincinnati
University of Dayton
University of Findlay
University of Toledo
Walsh University
Wilmington College
Wittenberg University
Wright State University
Xavier University

Oklahoma
Oklahoma State University
University of Oklahoma

Oregon
Lane Community College
Linfield College
Oregon State University
Pacific University
Southern Oregon State College
Southwestern Oregon Community College
University of Oregon

Pennsylvania
Art Institute of Pittsburgh
California University of Pennsylvania
Chatam College
Culinary Institute of Pittsburgh
Indiana University of Pennsylvania
Pennsylvania State University
Point Park College
Waynesburg College

South Carolina
Aiken Technical College
Charleston Southern University
Clemson University
College of Charleston
Denmark Technical College
Furman University
Horry-Georgetown Technical College
Lander University
Newberry College
North Greenville College
The Citadel
University of South Carolina
University of South Carolina, Aiken
Winthrop University

South Dakota
National American University

Tennessee
Belmont University
Maryville College
Tennessee Technological University
Trevecca Nazarene University
University of Tennessee
Vanderbilt University

Texas
Rice University
Texas A&M University-Corpus Christi
Texas Junior College

Utah
Brigham Young University
University of Utah
Utah State University

Virginia
Hampton University
James Madison University
University of Virginia
University of Virginia, Richmond
Washington and Lee University

Washington
University of Puget Sound
Western Washington University

Washington, D.C.
American University
George Washington University

Wisconsin
Saint Norbert College
University of Wisconsin, Madison
University of Wisconsin, Milwaukee

Wyoming
University of Wyoming

International
American College, London, England
College in Japan
Kings Bible College, Scotland

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